

FINITE ELEMENT STUDY OF ACOUSTIC WAVES

by

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United States Naval Postgraduate School



THESIS

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December 1970

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Finite Element Study of Acoustic Waves

by

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ABSTRACT

The generation and propagation of small amplitude acoustic waves in a homogeneous, loss-free, compressible fluid is studied by the finite element method. A diaphragm mounted in an infinite rigid baffle generates acoustic waves in a semi-infinite fluid region. Steady-state pressure distribution is found for a hemispherical region with boundary reflection suppressed through use of a radiation condition. The computer program developed for the purpose utilizes iso-parametric finite elements with curvilinear boundaries. Incorporated in the program is a versatile mesh generator which minimizes the quantity of input data. Acceptable agreement with analytic results is obtained when there are at least four elements per wave-length.

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I. INTRODUCTION

The field of acoustics plays an important role in the development of the complex sonar systems used in the United States Navy today. It is the intent of this paper to open a new avenue to the study of the active sonar system. Attention is confined to the transducer and the periodic pressure field induced in a fluid medium by harmonic oscillations of the transducer piston or diaphragm.

The study is restricted to the steady-state performance of a transducer mounted in an infinite rigid baffle and radiating into a half-space filled with a homogeneous fluid. Both the pressure field and the acoustic impedance of the transducer are determined. Figure 1 illustrates the geometry of the region.

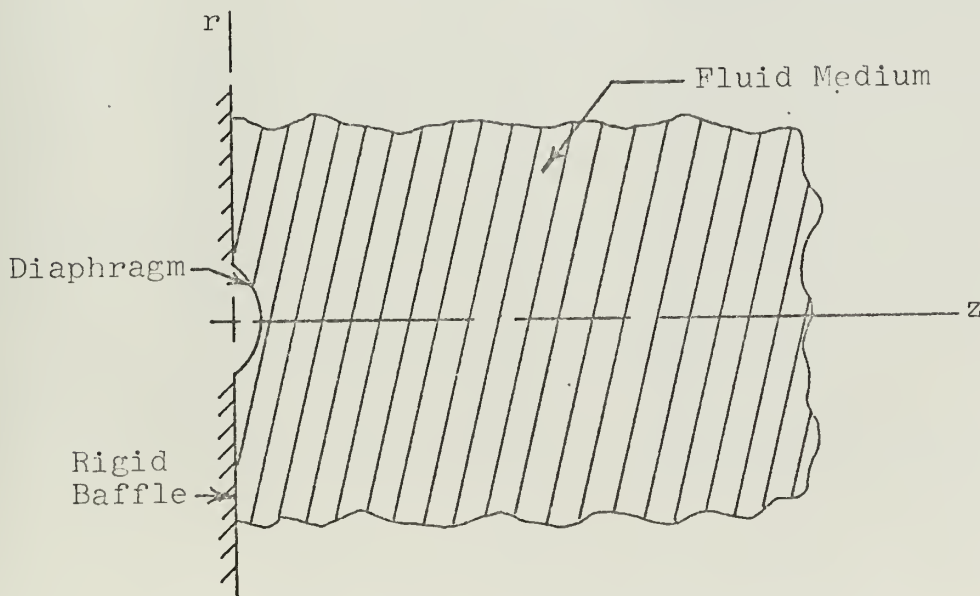


Figure 1. Geometry of Region.

The method used in this study is the two-dimensional finite element technique. Since it is manifestly impossible to represent an infinite region with a finite number of finite elements, the stratagem of the anechoic chamber is simulated mathematically. This is accomplished by using radiation (non-reflection) boundary conditions.

The diaphragm, being circular, forms a surface of revolution when deflected. Thus, the instantaneous pressure distribution has rotational symmetry about an axis through the diaphragm center and normal to the rigid baffle (z axis of Figure 1). A finite element assembly representing the region of interest may be constructed by dividing the corresponding portion of the r - z plane of Figure 1 into sub-areas as shown in Figure 2.

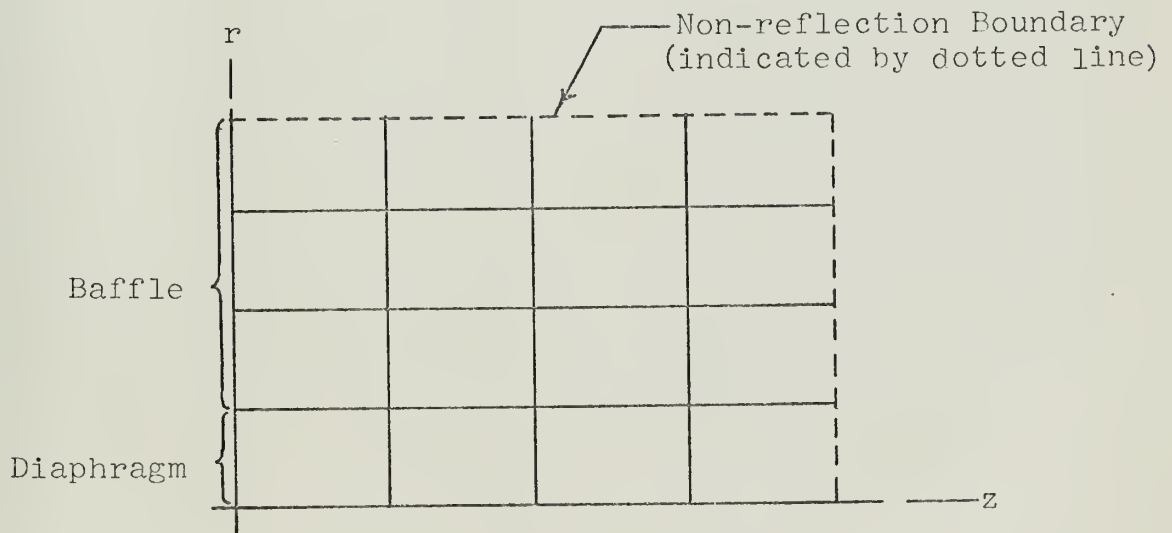


Figure 2. Finite Element Grid

In Figure 2, an individual rectangular element represents the cross-section of a toroidal element generated by revolution about the z axis. The rotational symmetry permits calculation of element properties using two-dimensional technique by simply taking the thickness (normal to the plane of the figure) proportional to r .

II. FINITE ELEMENT FORMULATION OF THE GOVERNING EQUATION

A. EQUATION OF FLUID MOTION

The governing partial differential equation for the pressure p in a homogeneous, inviscid, compressible fluid as used in acoustics (e.g., see Ref. [1]^{*}) is

$$\nabla^2 p = \frac{1}{c^2} \ddot{p} \quad (1)$$

where ∇^2 is the Laplacian operator, c is the acoustic velocity, and superior dots represent differentiation with respect to time. For the purpose of the present study, boundary conditions take the form

$$\frac{\partial p}{\partial n} = -\rho \dot{v}_n \quad (2)$$

where the coordinate n is measured along the normal to the boundary surface, ρ is the fluid density, and v_n is the component of fluid particle velocity in the n direction.

B. SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS

The finite element method replaces Eqs. (1) and (2) by a set of simultaneous ordinary differential equations with independent variable time and dependent variables consisting of pressures at a large number of discrete points (nodes) of the region. The region is divided into a number of

^{*}Numbers in brackets designate references listed on page 68.

sub-regions called "elements." Within each element it is postulated that the pressure p may be expressed in terms of the pressure p_i at the n nodes of the element. This relation is expressed as

$$p = \sum_{i=1}^n N_i p_i \quad (3)$$

where the "shape" (or interpolation) functions N_i are functions of the spatial coordinates. If the nodes are located at suitably chosen points on the element surface, unique sets of shape functions will guarantee the necessary continuity of pressure p along interelement boundaries as pointed out by Zienkiewicz [2]. The elements used in this study are two-dimensional eight-noded iso-parametric elements. Further details are given in Appendix C.

It has been shown by Zienkiewicz and Newton [3] that the finite element equation for k nodes, m of which are located on the fluid-diaphragm interface, may be written as

$$[Q]\{\ddot{p}\} + [D]\{\dot{p}\} + [H]\{p\} = - \rho [L]^T \{\ddot{\delta}\} \quad (4)$$

where $\{p\}$ is a $k \times 1$ vector of nodal pressures, $[Q]$, $[D]$, and $[H]$ are $k \times k$ symmetric matrices, $\{\delta\}$ is an $m \times 1$ vector of diaphragm displacements, and $[L]^T$ is a $k \times m$ matrix. Elements of $[Q]$, $[D]$, $[H]$, and $[L]^T$ are all real constants. Formulas for calculating these matrices are given in Appendix E. The terms $[D]\{\dot{p}\}$ and $[L]^T \{\ddot{\delta}\}$ represent boundary conditions and will be discussed in following sections.

C. DIAPHRAGM BOUNDARY CONDITION

The diaphragm displacement δ is a prescribed function of time. It can be represented by choosing a number of points (nodes) on the diaphragm and using a set of shape functions N_j' to interpolate the local displacement as

$$\delta = \sum_{j=1}^m N_j' \delta_j \quad (5)$$

where the δ_j are the displacements at the m diaphragm nodes.

The boundary condition of Eq. (2) is formulated in terms of the normal component v_n of fluid particle velocity. At the diaphragm surface this must, in the absence of cavitation, be equal to $\dot{\delta}$. Using this equality, the fluid-diaphragm interface condition gives the term

$$-\rho [L]^T \{\ddot{\delta}\}$$

in the finite element equation (Eq. (4)). The resulting vector has nonzero elements only for those pressure nodes located on the diaphragm surface. It is noted that the boundary condition on the rigid baffle is a special case, in that the particle velocity $v_n = 0$, which implies $\dot{v}_n = 0$.

D. RADIATION (NON-REFLECTION) BOUNDARY CONDITION

A normally incident wave meeting the interface between two media will not be reflected if the specific acoustic impedances of the two media are equal. The specific acoustic impedance z is defined to be the ratio of pressure to

particle velocity. For plane harmonic waves this is a real quantity and the pressure can be given as

$$p = \rho c v_n \quad (6)$$

for a wave progressing in the positive n direction. Differentiating Eq. (6) with respect to time and using this result to eliminate \dot{v}_n from the boundary condition (Eq. (2)) gives

$$\frac{\partial p}{\partial n} = - \frac{1}{c} \dot{p} \quad (7)$$

Equation (7) then becomes the boundary condition for non-reflection of normally incident plane waves. The term

$$[D]\{\dot{p}\}$$

in Eq. (4) results from this criterion. This vector has nonzero elements only for those nodes which lie on the non-reflecting portion of the boundary.

III. GEOMETRY OF THE REGION

The initial solution of the problem using the geometric configuration of Figure 2 resulted in significant error in the pressure distribution when the solution was checked against known results. The principal cause of this error was believed to be the shape of the non-reflection boundary. By the nature of the geometry, the harmonic oscillating diaphragm will create progressive pressure waves which propagate outward and strike the boundary of Figure 2 at various angles of incidence. The particle velocity, v_n , will not always be normal to the non-reflection boundary and the boundary condition required by Eq. (2) will not be satisfied. Thus, a modification to the representation of the boundary to provide substantially normal incidence led to the geometry illustrated in Figure 3. For this modified radiation boundary, in three dimensions the surface is hemispherical.

In order to represent adequately the circular arc of the non-reflection boundary, two-dimensional iso-parametric finite elements are used. The element thickness normal to the plane of the figure is directly proportional to the radius r . Iso-parametric elements are further discussed in Appendix C.

Note the distortion of the grid in Figure 3. The reason for this distortion and the method of generating the grid

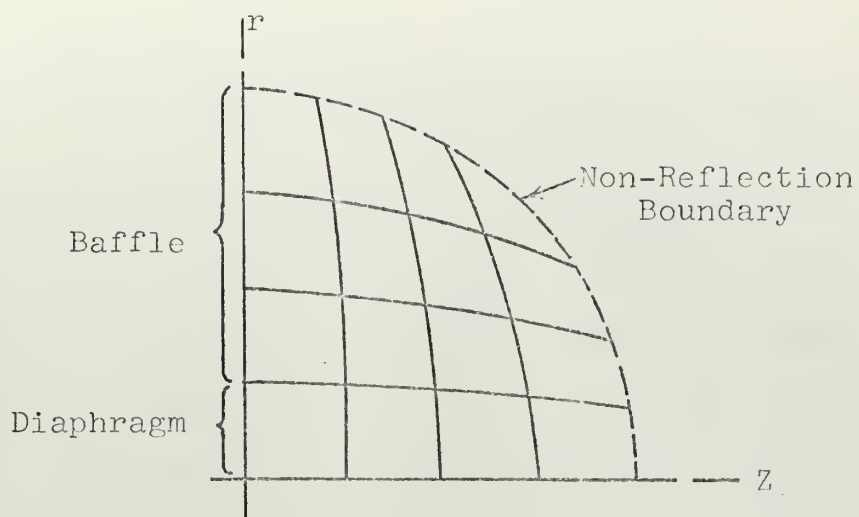


Figure 3. Modified Finite Element Region

pattern is discussed in Appendix D. The distorted "rectangles" still represent the cross sections of toroidal elements.

IV. WAVE THEORY AT RADIATION BOUNDARY

A one-dimensional study of a pulsating sphere acting on a fluid medium resulted in a more accurate solution when spherical wave theory at the non-reflection boundary was introduced. Based upon the success of these results, a further modification was made to the computer program (Appendix B) to incorporate spherical wave theory in the two-dimensional study. This was accomplished by developing the non-reflection boundary condition for spherically diverging waves as shown below.

The specific acoustic impedance for spherically diverging waves is given in Ref. [1] as

$$Z = \frac{p}{v_n} = \rho c \frac{\frac{\omega}{c} r}{\frac{\omega}{c} r - j} \quad (8)$$

where ω is the circular frequency and r is the radial distance from the "center." The particle acceleration \dot{v}_n can be written as

$$\dot{v}_n = j\omega v_n \quad (9)$$

which allows the boundary condition to be represented by

$$\frac{\partial p}{\partial r} = - \rho j\omega v_n \quad (10)$$

From Eqs. (8) and (10), the boundary condition is

$$\frac{\partial p}{\partial r} = - \frac{j \omega p}{c} \left[\frac{\frac{\omega}{c} r - j}{\frac{\omega}{c} r} \right] \quad (11)$$

or

$$\frac{\partial p}{\partial r} = - p \left(\frac{1}{r} + j \frac{\omega}{c} \right) \quad (12)$$

The imaginary term on the right hand side of Eq. (12) is represented by the term $[D]\{\dot{p}\} = j\omega[D]\{p\}$ in Eq. (4). Thus, the contribution of the real term can be written as

$$\frac{c}{r}[D]\{p\} \quad (13)$$

and can be added to the system of equations given in Eq. (4).

The spherical contribution is only added to the nodes which lie on the non-reflection boundary, thus providing a spherical wave approximation to the plane wave theory at the non-reflection boundary.

V. COMPUTER SOLUTION TECHNIQUE

Assume the diaphragm is oscillating with a harmonic displacement represented in the complex form by

$$\{\delta\} = \{\delta_o\}e^{j\omega t} \quad (14)$$

where $\{\delta_o\}$ is a vector of prescribed displacement amplitudes at the m nodes of the diaphragm-fluid interface. Elements of $\{\delta_o\}$ are real. The pressure in the fluid medium is then represented by

$$\{p\} = \{p_o\}e^{j\omega t} \quad (15)$$

where $\{p_o\}$ is a $k \times 1$ vector of pressure amplitudes (complex) at the k nodes of the region.

Inserting Eqs. (14) and (15) into Eq. (4) gives

$$([H] - \omega^2[Q] + j\omega[D])\{p\} = \omega^2\{B\} \quad (16)$$

where $\{B\} = [L]^T\{\delta_o\}$.

In the computer program, the system matrices $[H]$, $[Q]$, $[D]$, and $\{B\}$ are assembled from the contributions of each individual finite element in various subroutines. Since only a small number of nodes are located on the non-reflection boundary, the matrix $[D]$ is sparse. In order to reduce computer storage requirements, $[D]$, which is symmetric, is stored as three vectors: the principal diagonal and the two diagonals immediately above. At the same time, the approximated spherical contribution $\frac{c}{r}[D]$ at the

non-reflection boundary is computed. These quantities are real and are added directly to the matrix [H] in the appropriate locations.

The final subroutine introduces the value of omega and multiplies the matrices [Q], [D], and {B} by the appropriate power of omega before the left-hand side is summed to form one complex matrix. Thus the system of equations is reduced to

$$[S]\{p\} = \{F\} \quad (17)$$

where [S] is a $k \times k$ complex matrix and {F} is a column vector of order $k \times 1$ in which only m elements are nonzero.

The complex pressure amplitudes in Eq. (17) are found by using the Gaussian elimination and back substitution method.

The absolute value of the pressure and the phase angle are calculated in the program and printed out in array form. Additionally, a system energy balance and the diaphragm acoustic impedance are determined.

The program provides for multiple-problem solution by using incremented values of omega so that a range of frequencies can be studied.

VI. RESULTS

Problems studied were chosen to verify the accuracy of the method through comparison with available, exact (analytic) solutions. Results for each problem are discussed below.

Problem 1. Piston in a Rigid Tube

To verify the correctness of the computer program, finite element solutions were obtained for the problem of a harmonically oscillating circular piston at one end of a fluid-filled, rigid pipe of equal diameter. An infinite length was simulated by terminating the finite element region with a plane non-reflection boundary at a distance of one piston diameter. This is, of course, a one dimensional problem with uniform pressure amplitude throughout and phase angle is a linear function of distance along the pipe.

Detailed results are not presented for this essentially trivial test problem. It was found, however, that there was excellent agreement with theory for long wave-lengths. For wave-lengths less than approximately four times the element width (measured parallel to the direction of wave propagation), the accuracy deteriorated. This is understandable, since the shape functions employed represent the instantaneous pressure profile across an element by a parabolic arc. The lower limit on wave-length corresponds,

for a given element grid, to an upper limit on frequency.

Problem 2. Piston in an Infinite Baffle

The problem explored most extensively is that of a harmonically oscillating circular piston surrounded by an infinite rigid baffle and radiating into a semi-infinite fluid region. For this problem, the radiation boundary is a hemispherical surface with the center located at the piston center and the radius is four times that of the piston. The element grid (see Figure 4, Appendix A) contains 16 elements. It is derived by mapping a 4×4 square array from the ξ, η plane into a quadrant of a circle.

Analytic results for this problem are readily available (e.g., see [1], [4]). In Table I, the pressure amplitudes and phase angles along the z axis, obtained by the finite element method (FEM), are compared with exact results. Two sets of finite element results are given. For those designated FEM1, the spherical wave correction at the radiation boundary is included. For FEM2, the correction is omitted. Details are given in Table I for two circular frequencies, $\omega = .1$ and $.6$ rad/sec. The corresponding wave-lengths are approximately 63 ft and 10 feet.

Examination of Table I shows that FEM1 results give generally good agreement with exact results. Those for FEM2 are considerably poorer, although the discrepancies are smaller for $\omega = .6$ rad/sec. This is understandable since the $1/r$ contribution to specific acoustic impedance is relatively less important as ω increases (see Eq. (12)).

It is believed that a more refined finite element mesh (more elements) should afford significant improvement. Since the computer program for this problem required 320K bytes of core storage, it was not feasible to investigate this without major program revisions.

In predicting transducer performance the resultant fluid reaction on the piston is needed. This is normally reported in the form of piston radiation impedance, which is complex and is found by dividing the piston force by the piston velocity. The real part of the radiation impedance is called the radiation resistance and the imaginary part is called the radiation reactance.

Table II provides a comparison with exact values for the radiation resistance and reactance. Results are given for FEM1 and FEM2 over a range of frequencies. Again it is observed that the spherical correction significantly improves the results. Note that the radiation resistance obtained by FEM1 continues to give good results through $\omega = .9$ rad/sec.

Computer solutions of this problem were obtained for a range of frequency from $\omega = .1$ rad/sec to $\omega = 1.2$ rad/sec by .1 rad/sec increments. Comparisons at the upper end of this range are omitted from Tables I and II because the departure from exact results does not permit useful employment of the finite element method without grid refinement. Complete computer results for pressure amplitude, phase

TABLE I
PRESSURE AMPLITUDE AND PHASE ANGLE ON AXIS

| ω
rad/sec | z
ft | PRESSURE AMPLITUDE | | | PHASE ANGLE | | |
|---------------------|---------|-----------------------------|----------------------------|----------------------------|--------------|-------------|-------------|
| | | EXACT
lb/ft ² | FEM1
lb/ft ² | FEM2
lb/ft ² | EXACT
deg | FEM1
deg | FEM2
deg |
| .1 | 0 | .01997 | .02061 | .01794 | -5.7 | -5.7 | -11.6 |
| .1 | 2 | .00828 | .00889 | .00676 | -13.8 | -13.1 | -31.9 |
| .1 | 4 | .00472 | .00469 | .00376 | -24.3 | -24.9 | -68.3 |
| .1 | 6 | .00325 | .00329 | .00337 | -35.3 | -35.8 | -92.1 |
| .1 | 8 | .00246 | .00247 | .00334 | -46.5 | -47.4 | -107.3 |
| .6 | 0 | .678 | .699 | .625 | -34.4 | -33.2 | -34.7 |
| .6 | 2 | .295 | .300 | .273 | -83.0 | -78.7 | -89.2 |
| .6 | 4 | .169 | .172 | .186 | -145.6 | -145.4 | -150.5 |
| .6 | 6 | .117 | .116 | .106 | -211.8 | -211.0 | -210.6 |
| .6 | 8 | .089 | .086 | .084 | -279.3 | -279.2 | -289.3 |

TABLE II
PISTON RADIATION RESISTANCE AND REACTANCE

| ω
rad/sec | RESISTANCE | | | REACTANCE | | |
|---------------------|---|--|--|---|--|--|
| | EXACT
$\frac{\text{lb. sec}}{\text{ft}}$ | FEM1
$\frac{\text{lb. sec}}{\text{ft}}$ | FEM2
$\frac{\text{lb. sec}}{\text{ft}}$ | EXACT
$\frac{\text{lb. sec}}{\text{ft}}$ | FEM1
$\frac{\text{lb. sec}}{\text{ft}}$ | FEM2
$\frac{\text{lb. sec}}{\text{ft}}$ |
| .1 | .249 | .254 | .451 | 2.11 | 2.06 | 1.68 |
| .2 | .979 | .978 | .687 | 4.09 | 3.97 | 3.56 |
| .3 | 2.13 | 2.12 | 1.43 | 5.81 | 5.65 | 5.83 |
| .4 | 3.61 | 3.62 | 3.75 | 7.18 | 6.99 | 8.11 |
| .5 | 5.32 | 5.34 | 6.75 | 8.13 | 7.87 | 7.43 |
| .6 | 7.12 | 7.12 | 6.61 | 8.62 | 8.31 | 7.00 |
| .7 | 8.89 | 8.87 | 7.50 | 8.67 | 8.35 | 8.72 |
| .8 | 10.51 | 10.53 | 10.89 | 8.32 | 7.99 | 9.59 |
| .9 | 11.90 | 11.97 | 13.60 | 7.64 | 7.24 | 6.69 |

FEM1 - Finite element results with spherical correction.
FEM2 - Finite element results without spherical correction.

Phase angle given is for pressure relative to piston displacement.

Fluid density = 1 lb.sec/ft⁴

Piston Radius = 2 ft.

Acoustic Velocity = 1 ft/sec.

Piston Amplitude = 1 ft.

angle, and impedance components at several frequencies are given in Appendix A.

Problem 3. Parabolic Diaphragm in an Infinite Baffle

This problem, a modification of the previous problem, replaces the piston by a circular diaphragm having a fixed edge and having a deflection surface in the form of a paraboloid of revolution. Study of this problem was undertaken in an effort to improve agreement in the energy balance check (discussed below) incorporated into the program. It was believed that elimination of fluid velocity discontinuity at the edge of the piston in problem 1 might result in a significant improvement.

Analytic results for this problem are not available in standard acoustic texts, but conventional techniques can be utilized to obtain the pressure p along the z axis in complex form as

$$p = \frac{2j\rho c^3\delta_0}{\omega a^2} \left[\left(1 + j\frac{\omega s}{c} \right) e^{-j\frac{\omega s}{c}} - \left(1 + \frac{\omega^2 a^2}{2c^2} + j\frac{\omega z}{c} \right) e^{-j\frac{\omega z}{c}} \right] e^{j\omega t} \quad (18)$$

where

δ_0 = displacement amplitude at diaphragm center

a = diaphragm diameter

s = "slant" distance = $(z^2 + a^2)^{\frac{1}{2}}$

For this problem only a single finite element solution (FEM3) was obtained. The spherical correction is included. Table III gives a comparison with exact results (Eq. (18))

TABLE III
PRESSURE AMPLITUDE AND PHASE ANGLE ON AXIS

| ω
rad/sec | z
ft | PRESSURE AMPLITUDE | | PHASE ANGLE | |
|---------------------|---------|-----------------------------|----------------------------|--------------|-------------|
| | | EXACT
lb/ft ² | FEM3
lb/ft ² | EXACT
deg | FEM3
deg |
| .1 | 0 | .01322 | .01427 | -4.3 | -4.1 |
| .1 | 2 | .00438 | .00461 | -13.1 | -12.6 |
| .1 | 4 | .00240 | .00242 | -23.7 | -24.2 |
| .1 | 6 | .00164 | .00166 | -34.8 | -35.5 |
| .1 | 8 | .00124 | .00125 | -46.2 | -47.1 |
| .6 | 0 | .4598 | .4934 | -25.7 | -23.8 |
| .6 | 2 | .1565 | .1590 | -78.5 | -75.5 |
| .6 | 4 | .0864 | .0867 | -142.97 | -142.3 |
| .6 | 6 | .0589 | .0584 | -209.4 | -209.4 |
| .6 | 8 | .0445 | .0436 | -277.8 | -278.1 |

FEM3 - Finite element results with spherical correction.
Phase angle is for pressure relative to diaphragm displacement.
Fluid density = 1 lb.sec/ft⁴.
Acoustic velocity = 1 ft/sec.
Diaphragm radius = 2 ft.
Diaphragm amplitude (center) = 1 ft.

for pressure amplitudes and phase angles for values of
 $\omega = .1$ and $.6$ rad/sec.

Examination of Table III shows that agreement near the diaphragm (small values of z) is not as good as FEM1 in Table I. At the radiation boundary however, the FEM3 errors are comparable to those for FEM1.

Energy Balance Check

Prior experience, by others, suggested that an energy balance check would be a useful feature of a finite element program. Specifically, since solutions represent steady-state behavior, the input power, averaged over a cycle, should equal the corresponding average value of the output power. Input occurs at the piston, or diaphragm, and output at the radiation boundary. Expressions for these quantities are derived and incorporated in the computer program.

A number of program errors in the energy balance calculations (Appendix B) prevented valid results until the final stage of the investigation. Thus, the values of average power in and average power out are not included in the computer output (Appendix A). Recent computations, with program errors corrected, have shown excellent agreement of these quantities to at least six significant digits in the lower range of the circular frequency. Even at higher frequencies, where solution accuracy (pressure amplitudes and phase angles) deteriorates markedly, five significant digit agreement is maintained.

VII. CONCLUSIONS

It has been shown that the finite element method can provide acceptable results concerning generation and propagation of acoustic waves. The parabolic iso-parametric element provides an economical, accurate means of representing regions having curved boundaries.

The principal limitation of the method is evident from the empirically deduced results that, for acceptable accuracy, a single element must not span more than one-quarter wave-length. For wave-lengths of the order of magnitude of the piston radius, the number of nodes required would be around 900. For the present program, which already utilizes 320K bytes of core storage for 65 nodes, this is far beyond capability.

APPENDIX A COMPUTER OUTPUT

The rows of pressures and phase angles shown in the computer output correspond to the numbering system shown on the finite element region, Figure 4.

The zeros printed in the array of pressures and phase angles do not indicate computed values, but facilitate printing the output in array form.

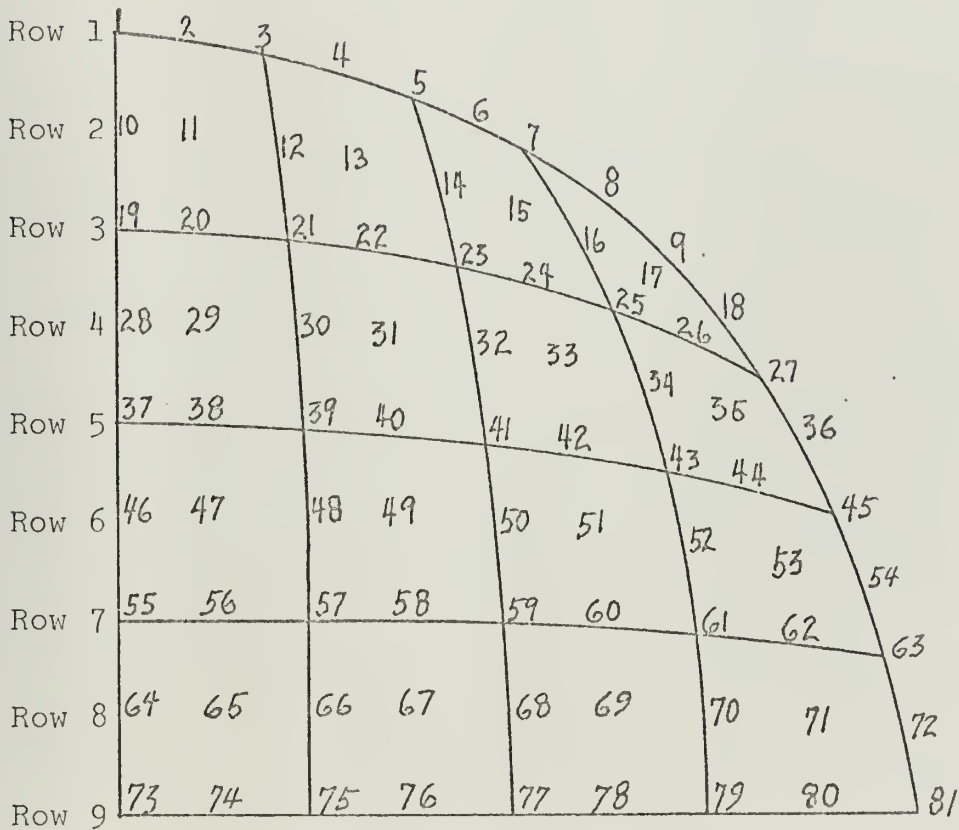


Figure 4. Format for Pressure and Phase Angle Array.

The output data are arranged according to the following sequence.

- FEM1. Piston in infinite baffle with spherical correction added at the non-reflection boundary.
- FEM2. Piston in infinite baffle without spherical correction added at the non-reflection boundary.
- FEM3. Diaphragm in infinite baffle with spherical correction added at the non-reflection boundary.

COMPUTER OUTPUT-GENERAL

TITLE OF GRAPH OF CORNER NODE POINTS TO BE PLOTTED
GRAPH ILLUSTRATING SYSTEM CORNER NODE LOCATIONS
FINITE ELEMENT STUDY OF ACOUSTIC WAVES...D.DEAN

NUMBER OF ELEMENTS=16
NUMBER OF NODES PER ELEMENT INCLUDING CENTER NODES= 9
NUMBER OF TOTAL SYSTEM NODES INCLUDING CENTER NODES=81
NUMBER OF NODES PER ELEMENT EXCLUDING CENTER NODES= 8
NUMBER OF SYSTEM NODES EXCLUDING CENTER NODES=65

| MAIN NODE NO | X | COORD | Y | COORD |
|--------------|---------|---------|---------|---------|
| 1 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | 4.0 | 0.0000 | 0.0 | 0.0 |
| 3 | 8.0 | 0.0000 | 0.0 | 0.0 |
| 4 | 7.39103 | 3.06146 | 3.06146 | 3.06146 |
| 5 | 5.65685 | 5.65685 | 5.65685 | 5.65685 |
| 6 | 3.06146 | 7.39103 | 7.39103 | 7.39103 |
| 7 | 0.0 | 8.00000 | 8.00000 | 8.00000 |
| 8 | 0.0 | 0.0 | 4.00000 | 4.00000 |

NUMBER OF ELEMENT DIVISIONS IN X DIRECTION= 4
NUMBER OF ELEMENT DIVISIONS IN THE Y DIRECTION= 4

ASSIGNMENT OF NODE NUMBERS TO ELEMENTS

| ELEMENT NO | NODE NUMBERS | (8 NODDED ELEMENTS) |
|------------|-------------------------|-------------------------|
| 1 | 15 16 17 18 19 20 21 22 | 3 5 7 9 11 12 13 14 |
| 2 | 17 18 19 20 21 22 23 24 | 5 7 9 11 12 13 14 15 |
| 3 | 19 20 21 22 23 24 25 26 | 7 9 11 12 13 14 15 16 |
| 4 | 21 22 23 24 25 26 27 28 | 9 11 12 13 14 15 16 17 |
| 5 | 23 24 25 26 27 28 29 30 | 11 12 13 14 15 16 17 18 |
| 6 | 25 26 27 28 29 30 31 32 | 13 14 15 16 17 18 19 20 |
| 7 | 27 28 29 30 31 32 33 34 | 15 16 17 18 19 20 21 22 |
| 8 | 29 30 31 32 33 34 35 36 | 17 18 19 20 21 22 23 24 |
| 9 | 31 32 33 34 35 36 37 38 | 19 20 21 22 23 24 25 26 |
| 10 | 33 34 35 36 37 38 39 40 | 21 22 23 24 25 26 27 28 |
| 11 | 35 36 37 38 39 40 41 42 | 23 24 25 26 27 28 29 30 |
| 12 | 37 38 39 40 41 42 43 44 | 25 26 27 28 29 30 31 32 |
| 13 | 39 40 41 42 43 44 45 46 | 27 28 29 30 31 32 33 34 |
| 14 | 41 42 43 44 45 46 47 48 | 29 30 31 32 33 34 35 36 |
| 15 | 43 44 45 46 47 48 49 50 | 31 32 33 34 35 36 37 38 |
| 16 | 45 46 47 48 49 50 51 52 | 33 34 35 36 37 38 39 40 |

COMPUTER OUTPUT-GENERAL

VALUES OF XI AND ETA FOR WHICH X AND Y COORDINATES WILL BE CALCULATED

VALUES OF XI

VALUES OF ETA

1 0.00000
2 -1.00000
3 -0.75000
4 -0.50000
5 -0.25000
6 0.00000
7 0.25000
8 0.50000
9 0.75000
1.00000

VALUES OF X AND Y

(X COORDINATES ON TOP-Y COORDINATES ON BOTTOM)

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 0.0 | 0.0 | 0.80906 | 1.58899 | 2.33979 | 3.06146 | 3.75400 | 4.41741 | 5.05170 | 5.65685 |
| 8.00000 | 8.00000 | 7.95325 | 7.83617 | 7.64876 | 7.39103 | 7.06297 | 6.66459 | 6.19588 | 5.65685 |
| 0.0 | 0.0 | 0.86369 | 1.70190 | 2.51462 | 3.30185 | 4.06359 | 4.79984 | 5.51061 | 6.19588 |
| 7.00000 | 7.00000 | 6.97183 | 6.88213 | 6.73090 | 6.51813 | 6.24382 | 5.90798 | 5.51061 | 5.05170 |
| 0.0 | 0.0 | 0.90954 | 1.79723 | 2.66307 | 3.50707 | 4.32922 | 5.12953 | 5.90798 | 6.66459 |
| 6.00000 | 6.00000 | 5.98678 | 5.92082 | 5.80211 | 5.63066 | 5.40647 | 5.12953 | 4.79984 | 4.41741 |
| 0.0 | 0.0 | 0.94659 | 1.87498 | 2.78516 | 3.67713 | 4.55090 | 5.40647 | 6.24382 | 7.06297 |
| 5.00000 | 5.00000 | 4.99809 | 4.95222 | 4.86240 | 4.72863 | 4.55090 | 4.32922 | 4.06359 | 3.75400 |
| 0.0 | 0.0 | 0.97486 | 1.93515 | 2.88087 | 3.81203 | 4.72863 | 5.63066 | 6.51813 | 7.39103 |
| 4.00000 | 4.00000 | 4.00575 | 3.97634 | 3.91177 | 3.81203 | 3.67713 | 3.50707 | 3.30185 | 3.06146 |
| 0.0 | 0.0 | 0.99433 | 1.97773 | 2.95021 | 3.91177 | 4.86240 | 5.80211 | 6.73090 | 7.64876 |
| 3.00000 | 3.00000 | 3.00978 | 2.99318 | 2.95021 | 2.88087 | 2.78516 | 2.66307 | 2.51462 | 2.33979 |
| 0.0 | 0.0 | 1.00501 | 2.00274 | 2.99318 | 3.97634 | 4.95222 | 5.92082 | 6.88213 | 7.83617 |
| 2.00000 | 2.00000 | 2.01016 | 2.00274 | 1.97773 | 1.93515 | 1.87498 | 1.79723 | 1.70190 | 1.58899 |
| 0.0 | 0.0 | 1.00690 | 2.01016 | 3.00978 | 4.00575 | 4.99809 | 5.98678 | 6.97183 | 7.95325 |
| 1.00000 | 1.00000 | 1.00690 | 1.00501 | 0.99433 | 0.97486 | 0.94659 | 0.90954 | 0.86369 | 0.80906 |
| 0.0 | 0.0 | 1.00000 | 2.00000 | 3.00000 | 4.00000 | 5.00000 | 6.00000 | 7.00000 | 8.00000 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

COMPUTER OUTPUT-GENERAL

NUMBER OF NODES ON NONREFLECTION BOUNDARY=17

NONREFLECTION BOUNDARY ALONG TOP OF ELEMENT IS FROM XI=-1.00 TO XI= 1.00

NONREFLECTION BOUNDARY ALONG RIGHT SIDE OF ELEMENT IS FROM ETA= 1.00 TO ETA=-1.00

NODE NUMBERS LOCATED ON THE NONREFLECTION BOUNDARY

| | | |
|---|----|----|
| 1 | 7 | 37 |
| 2 | 8 | 42 |
| 3 | 9 | 51 |
| 4 | 14 | 56 |
| 5 | 23 | 65 |
| 6 | 28 | |

SPEED OF SOUND = 1.00000

DENSITY OF FLUID= 1.00000

NUMBER OF NODES IN CONTACT WITH PISTON= 3

RANGE ON LEFT BOUNDARY WHICH PISTON IS IN CONTACT IS FROM ETA=-0.50 TO ETA=-1.00

NODE NUMBERS IN CONTACT WITH FORCING FUNCTION

43
52
57

AMPLITUDE OF PISTON AT THE INTERFACE NODES

| NODE | AMPLITUDE |
|------|-------------|
| 43 | 1.000000000 |
| 52 | 1.000000000 |
| 57 | 1.000000000 |

INITIAL VALUE OF OMEGA= 0.10000

INCREMENT VALUE OF OMEGA= 0.10000

NUMBER OF INCREMENTS OF OMEGA=12

FEM1. Piston in Infinite Baffle with Spherical Correction Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.10000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE
(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

| | | | | | | | | |
|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| 0.00253
-45.990 | 0.00254
-45.842 | 0.00253
-46.006 | 0.00253
-46.017 | 0.00252
-46.136 | 0.00252
-46.214 | 0.00251
-46.317 | 0.00251
-46.436 | 0.00250
-46.628 |
| 0.00290
-40.088 | 0.0
0.0 | 0.00386
-40.722 | 0.0
0.0 | 0.00276
-42.216 | 0.0
0.0 | 0.00263
-44.241 | 0.0
0.0 | 0.00250
-46.718 |
| 0.00341
-34.109 | 0.00334
-34.805 | 0.00328
-35.467 | 0.00316
-36.865 | 0.00302
-38.494 | 0.00289
-40.294 | 0.00275
-42.368 | 0.00262
-44.529 | 0.00249
-46.870 |
| 0.00405
-28.729 | 0.0
0.0 | 0.00380
-30.605 | 0.0
0.0 | 0.00334
-34.891 | 0.0
0.0 | 0.00287
-40.608 | 0.0
0.0 | 0.00249
-47.016 |
| 0.00509
-22.841 | 0.00501
-23.195 | 0.00451
-25.837 | 0.00411
-28.369 | 0.00369
-31.626 | 0.00331
-35.257 | 0.00299
-39.147 | 0.00272
-43.076 | 0.00249
-47.123 |
| 0.00733
-15.824 | 0.0
0.0 | 0.00565
-20.573 | 0.0
0.0 | 0.00403
-28.949 | 0.0
0.0 | 0.00309
-37.837 | 0.0
0.0 | 0.00249
-47.234 |
| 0.01232
-9.420 | 0.00907
-12.820 | 0.00669
-17.413 | 0.00532
-21.916 | 0.00437
-26.705 | 0.00369
-31.724 | 0.00318
-36.860 | 0.00280
-42.074 | 0.00250
-47.289 |
| 0.01858
-6.260 | 0.0
0.0 | 0.00777
-15.019 | 0.0
0.0 | 0.00463
-25.239 | 0.0
0.0 | 0.00324
-36.232 | 0.0
0.0 | 0.00251
-47.320 |
| 0.02061
-5.651 | 0.01175
-9.930 | 0.00889
-13.111 | 0.00621
-18.791 | 0.00469
-24.919 | 0.00385
-30.488 | 0.00329
-35.746 | 0.00282
-41.573 | 0.00247
-47.442 |

ACOUSTIC IMPEDANCE

REAL= 0.25423472
IMAG= 2.05667215

FEM1. Piston in Infinite Baffle with Spherical Correction Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.30000
ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE
(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

| | | | | | | | | |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 0.02178
-136.437 | 0.02174
-136.276 | 0.02177
-136.498 | 0.02174
-136.673 | 0.02174
-136.943 | 0.02174
-137.175 | 0.02175
-137.462 | 0.02176
-137.781 | 0.02177
-138.320 |
| 0.02481
-119.026 | 0.0
0.0 | 0.02450
-120.845 | 0.0
0.0 | 0.02378
-125.201 | 0.0
0.0 | 0.02284
-131.196 | 0.0
0.0 | 0.02180
-138.559 |
| 0.02897
-101.218 | 0.02876
-102.948 | 0.02808
-105.131 | 0.02722
-109.152 | 0.02620
-113.958 | 0.02502
-119.366 | 0.02398
-125.519 | 0.02287
-131.990 | 0.02181
-138.987 |
| 0.03496
-84.668 | 0.0
0.0 | 0.03290
-90.291 | 0.0
0.0 | 0.02897
-103.182 | 0.0
0.0 | 0.02509
-120.241 | 0.0
0.0 | 0.02184
-139.374 |
| 0.04404
-67.107 | 0.04289
-68.438 | 0.03933
-75.919 | 0.03572
-83.632 | 0.03213
-93.411 | 0.02893
-104.217 | 0.02620
-115.755 | 0.02384
-127.552 | 0.02184
-139.704 |
| 0.06255
-46.539 | 0.0
0.0 | 0.04889
-60.614 | 0.0
0.0 | 0.03547
-85.250 | 0.0
0.0 | 0.02719
-111.874 | 0.0
0.0 | 0.02194
-139.998 |
| 0.10619
-27.507 | 0.07915
-37.559 | 0.05916
-51.048 | 0.04714
-64.366 | 0.03866
-78.618 | 0.03255
-93.566 | 0.02803
-108.899 | 0.02462
-124.498 | 0.02203
-140.164 |
| 0.16362
-18.278 | 0.0
0.0 | 0.06930
-44.035 | 0.0
0.0 | 0.04110
-74.283 | 0.0
0.0 | 0.02862
-106.979 | 0.0
0.0 | 0.02212
-140.299 |
| 0.18304
-16.532 | 0.10585
-28.998 | 0.07853
-38.569 | 0.05498
-55.329 | 0.04205
-73.054 | 0.03434
-89.622 | 0.02888
-105.764 | 0.02486
-123.098 | 0.02181
-140.440 |

ACOUSTIC IMPEDANCE

REAL= 2.11780205
IMAG= 5.65370620

FEM1. Piston in Infinite Baffle with spherical Correction Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.60000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE

(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

| | | | | | | | | |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 0.07559
86.954 | 0.07613
86.875 | 0.07617
86.955 | 0.07693
86.417 | 0.07759
86.167 | 0.07835
85.708 | 0.07948
85.323 | 0.08040
84.812 | 0.08172
83.943 |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|

| | | | | | | | | |
|--------------------|------------|--------------------|------------|--------------------|------------|-------------------|------------|-------------------|
| 0.08637
121.203 | 0.0
0.0 | 0.08608
117.825 | 0.0
0.0 | 0.08540
109.550 | 0.0
0.0 | 0.08432
97.978 | 0.0
0.0 | 0.08282
83.676 |
|--------------------|------------|--------------------|------------|--------------------|------------|-------------------|------------|-------------------|

| | | | | | | | | |
|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|-------------------|-------------------|
| 0.09955
156.262 | 0.10061
154.261 | 0.09853
149.116 | 0.09735
141.666 | 0.09547
132.479 | 0.09262
121.592 | 0.09021
109.661 | 0.08687
96.811 | 0.08416
83.067 |
|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|-------------------|-------------------|

| | | | | | | | | |
|---------------------|------------|--------------------|------------|--------------------|------------|--------------------|------------|-------------------|
| 0.12283
-168.583 | 0.0
0.0 | 0.11745
179.914 | 0.0
0.0 | 0.10695
153.794 | 0.0
0.0 | 0.09566
120.244 | 0.0
0.0 | 0.08505
82.491 |
|---------------------|------------|--------------------|------------|--------------------|------------|--------------------|------------|-------------------|

| | | | | | | | | |
|---------------------|---------------------|---------------------|---------------------|--------------------|--------------------|--------------------|--------------------|-------------------|
| 0.15587
-133.372 | 0.14923
-137.699 | 0.14403
-150.744 | 0.13192
-167.172 | 0.12155
173.329 | 0.11097
152.061 | 0.10184
129.651 | 0.09312
106.141 | 0.08592
81.974 |
|---------------------|---------------------|---------------------|---------------------|--------------------|--------------------|--------------------|--------------------|-------------------|

| | | | | | | | | |
|--------------------|------------|---------------------|------------|---------------------|------------|--------------------|------------|-------------------|
| 0.21095
-93.934 | 0.0
0.0 | 0.17615
-122.376 | 0.0
0.0 | 0.13770
-170.201 | 0.0
0.0 | 0.10132
137.294 | 0.0
0.0 | 0.08690
81.489 |
|--------------------|------------|---------------------|------------|---------------------|------------|--------------------|------------|-------------------|

| | | | | | | | | |
|--------------------|--------------------|---------------------|---------------------|---------------------|--------------------|--------------------|--------------------|-------------------|
| 0.36675
-54.593 | 0.28723
-75.328 | 0.22673
-102.174 | 0.18449
-128.915 | 0.15359
-157.248 | 0.13028
173.449 | 0.11200
143.391 | 0.09811
112.580 | 0.08750
81.173 |
|--------------------|--------------------|---------------------|---------------------|---------------------|--------------------|--------------------|--------------------|-------------------|

| | | | | | | | | |
|--------------------|------------|--------------------|------------|---------------------|------------|--------------------|------------|-------------------|
| 0.60717
-36.413 | 0.0
0.0 | 0.27351
-88.602 | 0.0
0.0 | 0.16594
-148.976 | 0.0
0.0 | 0.11558
147.324 | 0.0
0.0 | 0.08829
80.855 |
|--------------------|------------|--------------------|------------|---------------------|------------|--------------------|------------|-------------------|

| | | | | | | | | |
|--------------------|--------------------|--------------------|---------------------|---------------------|---------------------|--------------------|--------------------|-------------------|
| 0.69899
-33.217 | 0.42606
-57.874 | 0.30000
-78.708 | 0.21718
-112.362 | 0.17233
-145.348 | 0.14117
-177.879 | 0.11589
148.974 | 0.09995
115.227 | 0.08645
80.824 |
|--------------------|--------------------|--------------------|---------------------|---------------------|---------------------|--------------------|--------------------|-------------------|

ACOUSTIC IMPEDANCE

REAL= 7.11911492

IMAG= 8.30592614

FEM1. Piston in Infinite Baffle with Spherical Correction Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.90000
ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE
(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

| | | | | | | | | |
|---------------------|---------------------|---------------------|---------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| 0.13355
-48.071 | 0.13613
-47.085 | 0.13579
-47.932 | 0.13968
-48.528 | 0.14245
-49.115 | 0.14650
-50.453 | 0.15200
-51.005 | 0.15661
-52.250 | 0.16354
-53.654 |
| 0.15353
4.213 | 0.0
0.0 | 0.15462
-1.440 | 0.0
0.0 | 0.15841
-14.777 | 0.0
0.0 | 0.16390
-32.507 | 0.0
0.0 | 0.16924
-54.061 |
| 0.17814
56.454 | 0.17905
54.050 | 0.17866
45.619 | 0.17980
33.786 | 0.18045
20.005 | 0.18019
2.857 | 0.18079
-14.822 | 0.17760
-34.395 | 0.17702
-54.733 |
| 0.22101
109.867 | 0.0
0.0 | 0.21494
91.552 | 0.0
0.0 | 0.20761
50.888 | 0.0
0.0 | 0.19663
0.578 | 0.0
0.0 | 0.18179
-55.698 |
| 0.28014
162.646 | 0.27153
153.347 | 0.26876
136.330 | 0.25401
109.466 | 0.24446
80.474 | 0.22875
47.889 | 0.21540
14.864 | 0.19977
-20.367 | 0.18712
-56.128 |
| 0.35583
-142.184 | 0.0
0.0 | 0.33268
175.666 | 0.0
0.0 | 0.28309
104.587 | 0.0
0.0 | 0.23312
25.969 | 0.0
0.0 | 0.19191
-56.922 |
| 0.63930
-79.378 | 0.54709
-111.828 | 0.46567
-151.062 | 0.38422
167.994 | 0.32637
124.652 | 0.28326
79.864 | 0.24769
35.119 | 0.21827
-10.397 | 0.19454
-57.420 |
| 1.20887
-53.483 | 0.0
0.0 | 0.59822
-131.342 | 0.0
0.0 | 0.36241
136.976 | 0.0
0.0 | 0.25955
40.911 | 0.0
0.0 | 0.19818
-57.750 |
| 1.46254
-49.631 | 0.97506
-85.219 | 0.63836
-118.876 | 0.48057
-168.541 | 0.37286
143.663 | 0.31430
93.398 | 0.26065
43.164 | 0.22686
-6.174 | 0.19239
-58.558 |

ACOUSTIC IMPEDANCE

REAL= 11.96803122
IMAG= 7.24443414

FEM2. Piston in Infinite Baffle Without Spherical Correction Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.10000
ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE
(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

| | | | | | | | | |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 0.00336
-104.805 | 0.00335
-104.666 | 0.00335
-104.877 | 0.00335
-104.952 | 0.00335
-105.151 | 0.00335
-105.318 | 0.00335
-105.517 | 0.00335
-105.741 | 0.00335
-106.040 |
| 0.00336
-98.047 | 0.0
0.0 | 0.00336
-98.930 | 0.0
0.0 | 0.00335
-100.924 | 0.0
0.0 | 0.00335
-103.435 | 0.0
0.0 | 0.00334
-106.234 |
| 0.00340
-88.884 | 0.00339
-90.120 | 0.00338
-91.277 | 0.00337
-93.588 | 0.00336
-96.099 | 0.00335
-98.663 | 0.00335
-101.366 | 0.00334
-103.939 | 0.00334
-106.480 |
| 0.00353
-77.901 | 0.0
0.0 | 0.00347
-82.123 | 0.0
0.0 | 0.00338
-90.469 | 0.0
0.0 | 0.00335
-99.231 | 0.0
0.0 | 0.00334
-106.708 |
| 0.00396
-62.364 | 0.00391
-63.401 | 0.00369
-70.759 | 0.00355
-77.149 | 0.00344
-84.376 | 0.00338
-91.204 | 0.00335
-97.300 | 0.00334
-102.433 | 0.00334
-106.878 |
| 0.00547
-40.418 | 0.0
0.0 | 0.00427
-55.516 | 0.0
0.0 | 0.00352
-78.541 | 0.0
0.0 | 0.00335
-95.428 | 0.0
0.0 | 0.00334
-107.024 |
| 0.00990
-21.154 | 0.00692
-31.017 | 0.00497
-45.494 | 0.00407
-59.634 | 0.00363
-73.085 | 0.00343
-84.648 | 0.00336
-93.930 | 0.00334
-101.277 | 0.00334
-107.089 |
| 0.01595
-13.011 | 0.0
0.0 | 0.00581
-37.842 | 0.0
0.0 | 0.00373
-69.199 | 0.0
0.0 | 0.00336
-92.929 | 0.0
0.0 | 0.00334
-107.128 |
| 0.01794
-11.564 | 0.00936
-22.554 | 0.00676
-31.885 | 0.00462
-49.896 | 0.00376
-68.341 | 0.00347
-82.022 | 0.00337
-92.074 | 0.00334
-100.682 | 0.00334
-107.263 |

ACOUSTIC IMPEDANCE

REAL= 0.45053255
IMAG= 1.68881084

FEM2. Piston in Infinite Baffle Without Spherical Correction Added at the Non-Reflection Boundary

| VALUE OF OMEGA FOR THIS RUN= 0.30000 | | | | | | | | | | | |
|--|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|--|--|--|
| ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE | | | | | | | | | | | |
| (PRESSURE ON TOP-PHASE ANGLE ON BOTTOM) | | | | | | | | | | | |
| 0.01798
-144.904 | 0.01793
-144.753 | 0.01796
-145.027 | 0.01793
-145.273 | 0.01792
-145.624 | 0.01790
-145.937 | 0.01788
-146.307 | 0.01785
-146.713 | 0.01784
-147.337 | | | |
| 0.01810
-125.114 | 0.0
0.0 | 0.01805
-127.452 | 0.0
0.0 | 0.01795
-132.844 | 0.0
0.0 | 0.01786
-139.799 | 0.0
0.0 | 0.01780
-147.655 | | | |
| 0.01959
-100.100 | 0.01949
-102.660 | 0.01909
-105.856 | 0.01871
-111.665 | 0.01835
-118.391 | 0.01806
-125.608 | 0.01791
-133.295 | 0.01781
-140.764 | 0.01779
-148.153 | | | |
| 0.02430
-75.714 | 0.0
0.0 | 0.02236
-83.713 | 0.0
0.0 | 0.01945
-102.782 | 0.0
0.0 | 0.01800
-126.700 | 0.0
0.0 | 0.01777
-148.599 | | | |
| 0.03414
-53.351 | 0.03293
-54.844 | 0.02866
-63.818 | 0.02485
-73.985 | 0.02153
-88.003 | 0.01932
-104.117 | 0.01821
-120.662 | 0.01781
-135.724 | 0.01775
-148.960 | | | |
| 0.05583
-33.246 | 0.0
0.0 | 0.03971
-46.246 | 0.0
0.0 | 0.02441
-75.995 | 0.0
0.0 | 0.01847
-115.134 | 0.0
0.0 | 0.01774
-149.232 | | | |
| 0.10358
-18.598 | 0.07417
-25.978 | 0.05134
-37.069 | 0.03723
-50.025 | 0.02759
-66.867 | 0.02167
-87.835 | 0.01876
-110.709 | 0.01784
-131.895 | 0.01772
-149.337 | | | |
| 0.16325
-12.217 | 0.0
0.0 | 0.06278
-31.053 | 0.0
0.0 | 0.03018
-61.282 | 0.0
0.0 | 0.01902
-107.811 | 0.0
0.0 | 0.01771
-149.440 | | | |
| 0.18311
-11.038 | 0.10276
-19.634 | 0.07324
-26.699 | 0.04636
-40.885 | 0.03115
-59.783 | 0.02309
-81.850 | 0.01908
-105.829 | 0.01784
-130.124 | 0.01773
-149.601 | | | |

ACOUSTIC IMPEDANCE

REAL= 1.42595320
IMAG= 5.82615666

FEM2. Piston in Infinite Baffle Without Spherical Correction Added at the Non-Reflection Boundary

| VALUE OF OMEGA FOR THIS RUN= 0.60000 | | | | | | | | | | | |
|--|---------------------|---------------------|---------------------|---------------------|---------------------|--------------------|--------------------|-------------------|--|--|--|
| ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE | | | | | | | | | | | |
| (PRESSURE ON TOP-PHASE ANGLE ON BOTTOM) | | | | | | | | | | | |
| 0.07266
75.250 | 0.07322
75.183 | 0.07325
75.287 | 0.07406
74.767 | 0.07471
74.563 | 0.07550
74.173 | 0.07662
73.887 | 0.07751
73.490 | 0.07886
72.755 | | | |
| 0.07526
113.624 | 0.0
0.0 | 0.07537
109.638 | 0.0
0.0 | 0.07604
100.072 | 0.0
0.0 | 0.07758
87.300 | 0.0
0.0 | 0.07984
72.632 | | | |
| 0.09072
155.462 | 0.09118
153.276 | 0.08809
147.519 | 0.08569
138.936 | 0.08312
128.045 | 0.08073
114.949 | 0.08012
100.845 | 0.07999
86.413 | 0.08122
72.166 | | | |
| 0.12570
-169.876 | 0.0
0.0 | 0.11660
179.760 | 0.0
0.0 | 0.09770
153.364 | 0.0
0.0 | 0.08352
113.852 | 0.0
0.0 | 0.08208
71.725 | | | |
| 0.16452
-140.455 | 0.15811
-144.033 | 0.15269
-154.753 | 0.13677
-168.533 | 0.11941
173.808 | 0.10150
151.879 | 0.08895
125.679 | 0.08323
97.443 | 0.08298
71.349 | | | |
| 0.20198
-105.068 | 0.0
0.0 | 0.18397
-131.383 | 0.0
0.0 | 0.14291
-171.060 | 0.0
0.0 | 0.03437
135.226 | 0.0
0.0 | 0.08370
71.063 | | | |
| 0.31905
-60.357 | 0.25874
-85.177 | 0.22424
-113.375 | 0.19598
-137.009 | 0.16406
-160.116 | 0.12955
174.279 | 0.10063
142.675 | 0.08637
105.627 | 0.08398
70.978 | | | |
| 0.53874
-38.377 | 0.0
0.0 | 0.25787
-99.898 | 0.0
0.0 | 0.17903
-153.381 | 0.0
0.0 | 0.10522
147.318 | 0.0
0.0 | 0.08446
70.782 | | | |
| 0.62510
-34.649 | 0.37118
-64.484 | 0.27339
-89.206 | 0.22279
-122.932 | 0.18621
-150.469 | 0.14495
-177.612 | 0.10611
149.403 | 0.08789
108.951 | 0.08400
70.736 | | | |

ACOUSTIC IMPEDANCE

REAL= 6.60598018
IMAG= 6.99861861

FEM2. Piston in Infinite Baffle Without Spherical Correction Added at the Non-Reflection Boundary

| VALUE OF OMEGA FOR THIS RUN= 0.90000 | | | | | | | | | | | |
|---|---------------------|---------------------|---------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--|--|--|
| ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE
(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM) | | | | | | | | | | | |
| 0.14431
-53.197 | 0.14672
-52.175 | 0.14637
-53.027 | 0.15017
-53.632 | 0.15273
-54.224 | 0.15675
-55.581 | 0.16209
-56.141 | 0.16658
-57.417 | 0.17368
-58.849 | | | |
| 0.15527
4.211 | 0.0
0.0 | 0.15551
-2.094 | 0.0
0.0 | 0.15868
-17.075 | 0.0
0.0 | 0.16676
-36.728 | 0.0
0.0 | 0.17922
-59.282 | | | |
| 0.19974
58.374 | 0.19974
56.119 | 0.19571
48.170 | 0.19144
36.540 | 0.18594
22.225 | 0.18015
3.375 | 0.17929
-16.747 | 0.17998
-38.681 | 0.18731
-59.978 | | | |
| 0.24969
105.800 | 0.0
0.0 | 0.24654
89.399 | 0.0
0.0 | 0.22950
53.048 | 0.0
0.0 | 0.19575
0.933 | 0.0
0.0 | 0.19235
-60.977 | | | |
| 0.28526
159.428 | 0.27934
149.237 | 0.28405
131.313 | 0.28240
105.178 | 0.27807
79.220 | 0.25105
49.966 | 0.21948
16.786 | 0.19803
-22.890 | 0.19796
-61.400 | | | |
| 0.38208
-140.381 | 0.0
0.0 | 0.33235
174.968 | 0.0
0.0 | 0.31326
100.366 | 0.0
0.0 | 0.24385
28.377 | 0.0
0.0 | 0.20267
-62.090 | | | |
| 0.71787
-83.015 | 0.62007
-111.521 | 0.49012
-148.131 | 0.37796
166.954 | 0.34305
119.408 | 0.31944
78.005 | 0.26506
37.436 | 0.21609
-11.265 | 0.20493
-62.402 | | | |
| 1.29487
-58.602 | 0.0
0.0 | 0.66050
-128.609 | 0.0
0.0 | 0.36737
132.192 | 0.0
0.0 | 0.28154
42.900 | 0.0
0.0 | 0.20818
-62.657 | | | |
| 1.55387
-54.747 | 1.10882
-88.011 | 0.72188
-117.150 | 0.48296
-165.799 | 0.37122
139.523 | 0.34985
89.706 | 0.28443
45.091 | 0.22566
-6.520 | 0.20580
-63.560 | | | |

ACOUSTIC IMPEDANCE

REAL= 13.60456934
IMAG= 6.68581093

FEM3. Diaphragm in Infinite Baffle with Spherical Contribution Added at the Non-Reflection Boundary

NUMBER OF INCREMENTS OF OMEGA=12

VALUE OF OMEGA FOR THIS RUN= 0.10000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE

(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

| | | | | | | | | |
|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| 0.00126
-46.066 | 0.00127
-46.009 | 0.00126
-46.082 | 0.00126
-46.125 | 0.00126
-46.201 | 0.00126
-46.253 | 0.00126
-46.325 | 0.00126
-46.401 | 0.00125
-46.553 |
| 0.00145
-40.277 | 0.0
0.0 | 0.00142
-40.861 | 0.0
0.0 | 0.00138
-42.263 | 0.0
0.0 | 0.00132
-44.206 | 0.0
0.0 | 0.00125
-46.607 |
| 0.00169
-34.393 | 0.00167
-34.906 | 0.00163
-35.646 | 0.00158
-36.926 | 0.00151
-38.500 | 0.00145
-40.263 | 0.00138
-42.288 | 0.00132
-44.416 | 0.00125
-46.729 |
| 0.00202
-28.783 | 0.0
0.0 | 0.00190
-30.623 | 0.0
0.0 | 0.00167
-34.859 | 0.0
0.0 | 0.00144
-40.492 | 0.0
0.0 | 0.00125
-46.834 |
| 0.00255
-22.849 | 0.00247
-23.542 | 0.00227
-25.719 | 0.00206
-28.296 | 0.00185
-31.592 | 0.00166
-35.131 | 0.00150
-38.961 | 0.00137
-42.883 | 0.00125
-46.928 |
| 0.00351
-16.563 | 0.0
0.0 | 0.00281
-20.740 | 0.0
0.0 | 0.00204
-28.695 | 0.0
0.0 | 0.00156
-37.633 | 0.0
0.0 | 0.00126
-47.002 |
| 0.00563
-10.325 | 0.00454
-12.815 | 0.00340
-17.136 | 0.00271
-21.541 | 0.00221
-26.433 | 0.00186
-31.460 | 0.00161
-36.599 | 0.00141
-41.822 | 0.00126
-47.042 |
| 0.01060
-5.490 | 0.0
0.0 | 0.00407
-14.325 | 0.0
0.0 | 0.00235
-24.849 | 0.0
0.0 | 0.00164
-35.925 | 0.0
0.0 | 0.00127
-47.070 |
| 0.01427
-4.084 | 0.00689
-8.472 | 0.00461
-12.642 | 0.00317
-18.450 | 0.00242
-24.176 | 0.00196
-29.994 | 0.00166
-35.525 | 0.00142
-41.336 | 0.00125
-47.116 |

ACOUSTIC IMPEDANCE

REAL= 0.06370248
IMAG= 0.63658158

FEM3. Diaphragm in Infinite Baffle with Spherical Contribution Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.30000
ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE
(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

| | | | | | | | | |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 0.01098
-136.759 | 0.01099
-136.686 | 0.01098
-136.789 | 0.01098
-136.962 | 0.01098
-137.141 | 0.01098
-137.291 | 0.01098
-137.484 | 0.01099
-137.691 | 0.01100
-138.120 |
| 0.01254
-119.497 | 0.0
0.0 | 0.01238
-121.205 | 0.0
0.0 | 0.01201
-125.339 | 0.0
0.0 | 0.01153
-131.107 | 0.0
0.0 | 0.01101
-138.254 |
| 0.01463
-101.915 | 0.01451
-103.289 | 0.01418
-105.574 | 0.01374
-109.335 | 0.01323
-113.990 | 0.01267
-119.283 | 0.01211
-125.309 | 0.01155
-131.681 | 0.01102
-138.590 |
| 0.01762
-84.934 | 0.0
0.0 | 0.01660
-90.420 | 0.0
0.0 | 0.01463
-103.097 | 0.0
0.0 | 0.01267
-119.916 | 0.0
0.0 | 0.01103
-138.881 |
| 0.02220
-67.248 | 0.02150
-69.403 | 0.01987
-75.697 | 0.01805
-83.460 | 0.01624
-93.262 | 0.01462
-103.847 | 0.01323
-115.263 | 0.01204
-127.029 | 0.01103
-139.149 |
| 0.03050
-48.708 | 0.0
0.0 | 0.02459
-61.047 | 0.0
0.0 | 0.01796
-84.587 | 0.0
0.0 | 0.01374
-111.289 | 0.0
0.0 | 0.01108
-139.369 |
| 0.04919
-30.231 | 0.03997
-37.565 | 0.03023
-50.239 | 0.02404
-63.295 | 0.01961
-77.811 | 0.01648
-92.797 | 0.01417
-108.173 | 0.01244
-123.813 | 0.01113
-139.491 |
| 0.09381
-16.037 | 0.0
0.0 | 0.03635
-42.030 | 0.0
0.0 | 0.02092
-73.145 | 0.0
0.0 | 0.01447
-106.142 | 0.0
0.0 | 0.01118
-139.605 |
| 0.12726
-11.928 | 0.06189
-24.759 | 0.04096
-37.151 | 0.02817
-54.245 | 0.02152
-71.068 | 0.01740
-88.328 | 0.01461
-105.049 | 0.01257
-122.397 | 0.01100
-139.656 |

ACOUSTIC IMPEDANCE

REAL= 0.54011493
IMAG= 1.80323806

FEM3. Diagram in Infinite Baffle with Spherical Contribution Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.60000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE

(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

| | | | | | | | | |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|--------------------|--------------------|-------------------|
| 0.04011
86.160 | 0.04037
86.264 | 0.04032
86.271 | 0.04061
85.924 | 0.04081
85.762 | 0.04104
85.464 | 0.04143
85.251 | 0.04170
84.916 | 0.04217
84.235 |
| 0.04589
120.534 | 0.0
0.0 | 0.04555
117.261 | 0.0
0.0 | 0.04478
109.265 | 0.0
0.0 | 0.04374
98.077 | 0.0
0.0 | 0.04252
84.154 |
| 0.05313
155.411 | 0.05328
153.344 | 0.05217
148.517 | 0.05113
141.252 | 0.04982
132.317 | 0.04806
121.693 | 0.04654
109.960 | 0.04460
97.299 | 0.04302
83.725 |
| 0.06475
-169.810 | 0.0
0.0 | 0.06160
179.198 | 0.0
0.0 | 0.05551
153.895 | 0.0
0.0 | 0.04907
120.789 | 0.0
0.0 | 0.04330
83.276 |
| 0.08137
-134.539 | 0.07902
-139.386 | 0.07479
-151.060 | 0.06843
-167.057 | 0.06284
173.856 | 0.05688
152.791 | 0.05193
130.427 | 0.04735
106.978 | 0.04360
82.908 |
| 0.11005
-98.250 | 0.0
0.0 | 0.09217
-122.886 | 0.0
0.0 | 0.07041
-169.083 | 0.0
0.0 | 0.05446
138.362 | 0.0
0.0 | 0.04393
82.499 |
| 0.17825
-60.629 | 0.14928
-75.537 | 0.11776
-100.607 | 0.09472
-126.915 | 0.07840
-155.495 | 0.06604
175.010 | 0.05658
144.687 | 0.04954
113.692 | 0.04418
82.261 |
| 0.35379
-32.026 | 0.0
0.0 | 0.14360
-84.780 | 0.0
0.0 | 0.08441
-146.616 | 0.0
0.0 | 0.05820
148.773 | 0.0
0.0 | 0.04453
81.997 |
| 0.49338
-23.844 | 0.24710
-49.582 | 0.15898
-75.523 | 0.11251
-109.603 | 0.08665
-142.253 | 0.07088
-175.848 | 0.05841
150.585 | 0.05035
116.504 | 0.04358
81.877 |

ACOUSTIC IMPEDANCE

REAL= 1.92789152

IMAG= 2.94996724

FEM3. Diaphragm in Infinite Baffle with Spherical Contribution Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.90000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE
(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

| | | | | | | | | |
|---------------------|---------------------|---------------------|---------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| 0.07730
-49.379 | 0.07845
-48.551 | 0.07809
-49.129 | 0.07959
-49.629 | 0.08049
-49.967 | 0.08183
-50.990 | 0.08384
-51.274 | 0.08522
-52.158 | 0.08779
-53.220 |
| 0.08865
2.627 | 0.0
0.0 | 0.08853
-2.737 | 0.0
0.0 | 0.08873
-15.424 | 0.0
0.0 | 0.08923
-32.433 | 0.0
0.0 | 0.08967
-53.302 |
| 0.10270
54.794 | 0.10326
52.062 | 0.10171
44.285 | 0.10117
32.852 | 0.09995
19.545 | 0.09814
2.927 | 0.09701
-14.369 | 0.09404
-33.615 | 0.09265
-53.661 |
| 0.12664
106.954 | 0.0
0.0 | 0.12139
89.798 | 0.0
0.0 | 0.11318
50.900 | 0.0
0.0 | 0.10389
1.436 | 0.0
0.0 | 0.09421
-54.370 |
| 0.15738
159.354 | 0.15560
151.303 | 0.14845
134.710 | 0.13865
109.309 | 0.13097
81.395 | 0.12056
49.091 | 0.11226
16.249 | 0.10323
-18.950 | 0.09610
-54.548 |
| 0.20929
-147.999 | 0.0
0.0 | 0.18441
175.456 | 0.0
0.0 | 0.14913
106.088 | 0.0
0.0 | 0.11988
27.836 | 0.0
0.0 | 0.09793
-55.160 |
| 0.33963
-89.983 | 0.29929
-112.628 | 0.24879
-149.068 | 0.20130
170.451 | 0.16988
127.379 | 0.14553
82.497 | 0.12613
37.463 | 0.11086
-8.422 | 0.09875
-55.516 |
| 0.72350
-47.294 | 0.0
0.0 | 0.31667
-126.271 | 0.0
0.0 | 0.18668
140.439 | 0.0
0.0 | 0.13143
43.543 | 0.0
0.0 | 0.10030
-55.774 |
| 1.05780
-35.353 | 0.55984
-73.512 | 0.34658
-113.561 | 0.25245
-163.877 | 0.18885
147.129 | 0.15994
96.461 | 0.13127
45.983 | 0.11438
-3.977 | 0.09734
-56.523 |

ACOUSTIC IMPEDANCE

REAL= 3.57037792
IMAG= 3.14224531

APPENDIX B - COMPUTER PROGRAM

THE FINITE ELEMENT STUDY OF ACOUSTIC WAVES CREATED BY A MECHANISM ACTING ON A FLUID SUBSTANCE AT THE CENTER OF AN INFINITE WALL. OTHER MECHANICAL MEANS OF APPLYING A FORCE TO THE FLUID. THE FLUID REGION AWAY FROM THE WALL IS CONSIDERED TO BE INFINITE.

THIS PROGRAM IS DESIGNED TO TAKE ONE 8 NODDED ELEMENT AND SUBDIVIDE IT INTO ANY NUMBER OF SMALLER ELEMENTS BY SPECIFYING THE NUMBER OF DIVISIONS DESIRED IN BOTH THE X AND Y DIRECTION. EACH SUBDIVIDED ELEMENT WILL AUTOMATICALLY BE ASSIGNED 9 GLOBAL NODE NUMBERS, 8 OF WHICH ARE ON THE BOUNDARY AND THE 9TH IS LOCATED AT THE CENTER. THE 9TH (CENTER) NODE IS USED TO PROVIDE A MEANS FOR FORMULATING LEGIBLE OUTPUT. COORDINATES OF ALL GLOBAL NODES ARE THEN CALCULATED AND PLOTTED TO FORM THE FINITE ELEMENT MESH FOR THE PROBLEM TO BE SOLVED. NON-REFLECTION BOUNDARY NODE NUMBERS AND THE NODE NUMBERS IN CONTACT WITH THE FORCING MECHANISM ARE AUTOMATICALLY ASSIGNED IN THE PROGRAM.

-----DATA CARDS FOR SUBROUTINE INPUT

- (1) TITLE OF GRAPH OF THE FINITE ELEMENT MESH WHICH WILL BE GENERATED AND PLOTTED IN THE PROGRAM.
(TWO CARDS) FORMAT(6A8) TITLE OF GRAPH
- (2) NUMBER OF FLUID ELEMENTS, NUMBER OF NODES PER ELEMENT AND TOTAL NUMBER OF SYSTEM NODES. AND EACH ELEMENT HAS 8 NODES ON THE BOUNDARY AND ONE MIDDLE NODE. THE MIDDLE NODE IS USED ONLY TO ASSIST IN PRINTING OUT THE RESULTS IN ARRAY FORM.
(ONE CARD) FORMAT(3I10,NE,MT,NNODT)
- (3) X AND Y COORDINATES OF THE EIGHT MAIN NODES ASSIGNED TO THE BOUNDARY OF THE REGION TO BE STUDIED. (ONE CARD PER NODE)
FORMAT(I2,2F10.5) K, XM(K), YM(K)
- (4) NUMBER OF SUBDIVISIONS REQUIRED OF THE TOTAL REGION IN BOTH THE X AND Y DIRECTION. (ONE CARD)
FORMAT(2I5) NXDIV, NYDIV
- (5) TOTAL NUMBER OF NODES LOCATED ON THE NON-REFLECTION BOUNDARY INCLUDING TWO VALUES OF XI DEFINING THE

RANGE OF THE NON-REFLECTION BOUNDARY ALONG THE
FACE ETA=1 AND THE TWO VALUES OF ETA WHICH DEFINE
THE RANGE OF THE NON-REFLECTION BOUNDARY ALONG
THE FACE XI=1. (ONE CARD)
FORMAT(I2,8X,4F10.5)NRBN,XI1,XI2,ETA1,ETA2

(6) SPEED OF SOUND AND THE FLUID DENSITY.
(ONE CARD) FORMAT(2F10.5)SOUND,FLDEN

(7) NUMBER OF NODES IN WHICH THE FORCING MECHANISM
IS IN CONTACT AND TWO VALUES OF ETA DEFINING THE
RANGE OF THE FORCING MECHANISM ACTING ON THE
FACE XI=-1. (ONE CARD)
FORMAT(I2,8X,2F10.5)NP,ETA1,ETA2

(8) AMPLITUDE OF THE FORCING FUNCTION AT THE NODES
WITH WHICH THE FORCING MECHANISM IS IN CONTACT.
AMPLITUDES READ FROM THE TOP DOWNWARD.
(ONE CARD) FORMAT(9F8.4)(AMP(I),I=1,NP)

(9) INITIAL VALUE OF OMEGA FOR WHICH PROBLEM IS TO
BE SOLVED; INCREMENT OF OMEGA FOR SUCCEEDING
SOLUTIONS; AND TOTAL NUMBER OF SOLUTIONS REQUIRED.
(ONE CARD) FORMAT(2F10.5,I2) OMEGA,OINCR,NO

CALL INPUT
CALL HQFORM
CALL FDAMP
CALL FORCE
CALL PROB
STOP
END

SUBROUTINE INPUT

IMPLICIT COMPLEX*16 (D),REAL*8 (A-C,E-H,O-Z)
REAL*8 ITITLE
COMMON DSUM(81,81),DB(81)
COMMON HS(81,81),Q(81,81),B(81),X(81),Y(81),HE(8,8),QE(8,8)
COMMON RD(50),RDS(50),RDSS(50),XM(8),YM(8),AMP(25)
COMMON GA(4),GH(4),SF(8),XD(8),YD(8),XDX(8),YDX(8),AJ(2,2),BL(3,3)
COMMON SOUND,FLDEN,OMEGA,OINCR
COMMON NN(100,8),NDAMP(50),NPN(50),NN9(8),NG(8),NN9(50,9),NDW(100)
COMMON NE,M,NNOD,NREN,NP,NO,NNODT,NC
COMMON /TITLE/ITITLE(12)


```

C
C
C
      TITLE OF GRAPH ON WHICH CORNER NODE POINTS WILL BE PLOTTED
      READ(5,142)(ITITLE(I),I=1,6)
      READ(5,142)(ITITLE(I),I=7,12)
      FORMAT(6A8)
142  WRITE(6,143)
143  FORMAT(11,'',TITLE OF GRAPH OF CORNER NODE POINTS TO BE PLOTTED '/')
144  WRITE(6,144) ITITLE
      FORMAT(4X,6A8/)
      NUMBER OF FLUID ELEMENTS, NODES PER ELEMENT, TOTAL
      NODES, AND NUMBER OF WORKING NODES.
      READ(5,100) NE,MT,NNODT
100  FORMAT(3I10)
      WRITE(6,101) NE
101  FORMAT(10,'',NUMBER OF ELEMENTS=',I2)
103  WRITE(6,103) MT
      FORMAT(10,'',NUMBER OF NODES PER ELEMENT INCLUDING CENTER NODES=',
112)
105  WRITE(6,105) NNODT
      FORMAT(10,'',NUMBER OF TOTAL SYSTEM NODES INCLUDING CENTER NODES=',
112)
106  M=MT-1
      WRITE(6,106) M
105  FORMAT(10,'',NUMBER OF NODES PER ELEMENT EXCLUDING CENTER NODES=',
112)
      NNOD=NNODT-NE
107  WRITE(6,107) NNOD
      FORMAT(10,'',NUMBER OF SYSTEM NODES EXCLUDING CENTER NODES=',I2)
      X AND Y COORDINATES FOR MAIN EIGHT NODES
      DO 13 I=1,NNODT
        X(I)=0. DO
        Y(I)=0. DO
13  WRITE(6,120)
120  FORMAT(10,'',MAIN NODE NO',6X,'X COORD',6X,'Y COORD')
      DO 12 I=1,M
        READ(5,121) K,XM(K),YM(K)
121  FORMAT(12,2F10.5)
12  WRITE(6,122) K,XM(K),YM(K)
122  FORMAT(5X,I2,11X,F10.5,3X,F10.5)
      ARRAY OF NODE NUMBERS AND FORMATION OF X AND Y COORDINATES.
C
C
C

```



```

170 READ(5,170)NXDIV,NYDIV
170 FORMAT(2I5)
171 WRITE(6,171) NXDIV
171 FORMAT('0',NUMBER OF ELEMENT DIVISIONS IN X DIRECTION=' ,I3)
172 WRITE(6,172) NYDIV
172 FORMAT('0',NUMBER OF ELEMENT DIVISIONS IN THE Y DIRECTION=' ,I3)
C
CALL NODE(NXDIV,NYDIV)
CALL XYFORM(NXDIV,NYDIV)
C
NUMBER OF NODES LOCATED ON THE NONREFLECTION BOUNDARY
AND THEIR RESPECTIVE NODE NUMBERS
C
114 READ(5,114) NRBN,XI1,XI2,ETA1,ETA2
114 FORMAT(I2,8X,4F10.5)
131 WRITE(6,131)NRBN
131 FORMAT('0',NUMBER OF NODES ON NONREFLECTION BOUNDARY=' ,I2)
115 WRITE(6,115) XI1,XI2
115 FORMAT('0',NONREFLECTION BOUNDARY ALONG TOP OF ELEMENT IS FROM
1 XI1=' ,F5.2,XI2=' ,F5.2)
116 WRITE(6,116) ETA1,ETA2
116 FORMAT('0',NONREFLECTION BOUNDARY ALONG RIGHT SIDE OF ELEMENT IS
1 FROM ETA=' ,F5.2,TO ETA=' ,F5.2)
XINCR=1.DO/NXDIV
YINCR=1.DO/NYDIV
NA=2*NXDIV*NYDIV+NXDIV+NYDIV+1
NB=NYDIV*(2*NXDIV+1)
NC=NXDIV
DO 10 I=1,NRBN
IF(XI1.EQ.XI2) GO TO 118
EN=NA-NB+NC*XI1
XI1=XI1+XINCR
IF(XI1.LT.XI2.OR.XI1.EQ.XI2) GO TO 20
118 EN=NA-NB*ETA1+NC
ETA1=ETA1-YINCR
II=EN
20 NDAMP(I)=NOW(II)
10 CONTINUE
132 WRITE(6,132)
132 FORMAT('0',NODE NUMBERS LOCATED ON THE NONREFLECTION BOUNDARY')
133 WRITE(6,133) (NDAMP(I),I=1,NRBN)
133 FORMAT(5X,I3/)
C
SPEED OF SOUND AND FLUID DENSITY
C
102 READ(5,102)SOUND,FLDEN
102 FORMAT(2F10.5)
WRITE(6,126) SOUND

```



```

126 FORMAT('0','SPEED OF SOUND =',F10.5)
WRITE(6,141)FLDEN
141 FORMAT('0','DENSITY OF FLUID=',F10.5)

      NUMBER OF FLUID NODES AND GLOBAL NODE NUMBERS WHICH
      FORCING MECHANISM IS IN CONTACT.

      READ(5,148)NP,ETAI,ETA2
148 FORMAT(12,8X,2F10.5)
      WRITE(6,149)NP
149 FORMAT('0','NUMBER OF NODES IN CONTACT WITH PISTON=',I4)
      WRITE(6,152)ETAI,ETA2
152 FORMAT('0','RANGE ON LEFT BOUNDARY WHICH PISTON IS IN CONTACT IS F
      1ROM ETA=',F5.2,1X,'TO ETA=',F5.2)
      WRITE(6,151)
151 FORMAT('0','NODE NUMBERS IN CONTACT WITH FORCING FUNCTION')
      DO 30 I=1,NP
      EN=NA-NB*ETAI-NC
      ETA1=ETAI-YINCR
      II=EN
      NPN(I)=NOW(II)
30 WRITE(6,153)NPN(I)
153 FORMAT(5X,I3)

      AMPLITUDE OF THE FORCING FUNCTION AT THE NODE POINTS
      WITH WHICH THE FORCING MECHANISM IS IN CONTACT.

      WRITE(6,138)
138 FORMAT('0','AMPLITUDE OF PISTON AT THE INTERFACE NODES')
      WRITE(6,139)
139 FORMAT('0','4X,'NODE',5X,'AMPLITUDE')
      READ(5,134)(AMP(I),I=1,NP)
134 FORMAT(9F8.4)
      WRITE(6,137)(NPN(I),AMP(I),I=1,NP)
137 FORMAT(6X,I2,2X,F16.8/)

      INITIAL VALUE OF OMEGA IN THE PISTON FORCING FUNCTION, INCREMENT VALUE
      OF OMEGA FOR SUCCEEDING SOLUTIONS, AND TOTAL NUMBER OF OMEGAS FOR
      WHICH PROBLEM IS TO BE CALCULATED

      READ(5,160) OMEGA,OINCR,NO
160 FORMAT(2F10.5,I2)
      WRITE(6,161) OMEGA
161 FORMAT('0','INITIAL VALUE OF OMEGA=',F10.5)
      WRITE(6,162) OINCR
162 FORMAT('0','INCREMENT VALUE OF OMEGA=',F10.5)
      NO
163 FORMAT('0','NUMBER OF INCREMENTS OF OMEGA=',I2)

```


RETURN
END

SUBROUTINE HQFORM

```

IMPLICIT COMPLEX*16 (D), REAL*8 (A-C, E-H, O-Z)
REAL*8 ITITLE
COMMON DSUM(81,81), DB(81)
COMMON H(81,81), Q(81,81), B(81), X(81), Y(81), HE(8,8), QE(8,8)
COMMON RD(50), RDS(50), RDSS(50), XM(8), YM(8), AMP(25)
COMMON GA(4), GH(4), SF(8), XD(8), YD(8), XXD(8), YYD(8), AJ(2,2), BL(3,3)
COMMON SOUND, FLDEN, OMEGA, QINCR
COMMON NN(100,8), NDAMP(50), NPN(50), NSF(8), NG(8), NN9(50,9), NOW(100)
COMMON NE, M, NNOD, NRB, NP, NO, NNODT, NC

```

SETTING ALL ELEMENTS OF Q AND H EQUAL TO ZERO

```

DO 20 I=1, NNOD
DO 20 J=1, NNOD
Q(I, J)=0.00
H(I, J)=0.00

```

20

GAUSS QUADRATURE ABSCISSA VALUES AND WEIGHT COEFFICIENTS

```

GA(1)= .861136311594053D0
GA(2)= .339981043584856D0
GA(3)= -.339981043584856D0
GA(4)= -.861136311594053D0
GH(1)= .347854845137454D0
GH(2)= .652145154862546D0
GH(3)= -.652145154862546D0
GH(4)= .347854845137454D0

```

FORMULATION OF THE ELEMENT HE AND QE MATRICES USING THE 4 TERM GAUSS QUADRATURE NUMERICAL INTEGRATION SCHEME

```

DO 21 L=1, NE
DO 22 I=1, M
DO 22 J=1, M
QE(I, J)=0.00
HE(I, J)=0.00

```

22

```

DO 23 LA=1, 4
DO 23 LB=1, 4
GX=GA(LA)
GY=GA(LB)
SF(1)= .25D0*(1.00-GX)*(1.00-GY)*(-GX-GY-1.00)

```



```

SF(2)=50D0*(1-D0-GX**2)*(1-D0-GY)*(1-D0-GY-1-D0)
SF(3)=25D0*(1-D0+GX)*(1-D0-GY)*(1-D0-GY-1-D0)
SF(4)=50D0*(1-D0+GX)*(1-D0-GY)**2*(1-D0-GY-1-D0)
SF(5)=25D0*(1-D0+GX)**2*(1-D0+GY)*(1-D0+GY-1-D0)
SF(6)=50D0*(1-D0-GX)*(1-D0-GY)*(1-D0+GY)*(-GX+GY-1-D0)
SF(7)=25D0*(1-D0-GX)*(1-D0-GY)**2*(1-D0-GY**2)
SF(8)=50D0*(1-D0-GY)*(1-D0-GY)*(1-D0-GY+GY)
XD(1)=-1-D0*(1-D0-GY)*(GX)
XD(2)=25D0*(1-D0-GY)**2*(2-D0*GX-GY)
XD(3)=50D0*(1-D0-GY)**2*(2-D0*GX+GY)
XD(4)=25D0*(1-D0+GY)*(GX)
XD(5)=-1-D0*(1-D0+GY)*(GX)
XD(6)=25D0*(1-D0+GY)**2*(2-D0*GX-GY)
XD(7)=-50D0*(1-D0-GY)**2*(2-D0*GY+GX)
XD(8)=25D0*(1-D0-GX)**2*(2-D0*GY-GX)
YD(1)=-1-D0*(1-D0-GX)**2*(2-D0*GX+GY)
YD(2)=50D0*(1-D0+GX)**2*(2-D0*GY-GX)
YD(3)=-1-D0*(1-D0+GX)*(GY)
YD(4)=25D0*(1-D0+GX)**2*(2-D0*GY+GX)
YD(5)=50D0*(1-D0-GX)**2*(2-D0*GX-GX)
YD(6)=-1-D0*(1-D0-GX)*(GY)
D0 24 I=1,2
D0 24 J=1,2
AJ(1,2)=0:D0
D0 24 I=1,M
AJ(1,1)=AJ(1,2)
AJ(1,2)=AJ(1,2)
AJ(2,1)=AJ(2,1)
AJ(2,2)=AJ(2,2)
AJ(1,1)*AJ(1,1)+XD(1)**X(NN(L,I))
AJ(1,2)*AJ(1,2)+XD(1)**Y(NN(L,I))
AJ(2,1)*AJ(2,1)+YD(1)**X(NN(L,I))
AJ(2,2)*AJ(2,2)+YD(1)**Y(NN(L,I))
ADET=AJ(1,1)*AJ(2,2)-AJ(1,2)*AJ(2,1)
TEMP=AJ(1,1)
AJ(1,1)=AJ(2,2)/ADET
AJ(1,2)=-AJ(1,2)/ADET
AJ(2,1)=-AJ(2,1)/ADET
AJ(2,2)=TEMP/ADET
D0 28 I=1,M
XXD(1)=AJ(1,1)*XD(1)+AJ(1,2)*YD(1)
YYD(1)=AJ(2,1)*XD(1)+AJ(2,2)*YD(1)
YY=0:D0
D0 29 I=1,M
YY=YY+SF(I)*Y(NN(L,I))
COEFH=ADEFH*GH(LB)*YY
COEFQ=COEFH/SOUND**2
D0 23 LC=1,M
D0 23 LD=1,M
FUNH=COEFH*(XXD(LC)*XXD(LD)+YYD(LC)+YYD(LD))

```



```

      FUNQ=COEFQ*SF(LC)*SF(LD)
      QE(LC,LD) = QE(LC,LD) + FUNQ
23  HE(LC,LD) = HE(LC,LD) + FUNH
C
C
      ADDING ELEMENT HE AND QE MATRICES TO TOTAL FLUID MATRICES
25  DO 21 I=1,M
      DO 21 J=1,M
      II=NN(L,I)
      JJ=NN(L,J)
21  Q(II,JJ)=Q(II,JJ)+QE(I,J)
      H(II,JJ)=H(II,JJ)+HE(I,J)
      RETURN
      END
C
      SUBROUTINE FDAMP
      IMPLICIT COMPLEX*16 (D),REAL*8 (A-C,E-H,O-Z)
      REAL*8 ITITLE
      COMMON DSUM(81,81),DB(81)
      COMMON H(81,81),Q(81,81),B(81),X(81),Y(81),HE(8,8),QE(8,8)
      COMMON RD(50),RDS(50),RDSS(50),XM(8),YM(8),AMP(25)
      COMMON GA(4),GH(4),SF(8),XD(8),YD(8),XVD(8),AJ(2,2),BL(3,3)
      COMMON SOUND,FLDEN,OMEGA,QINCR
      COMMON NN(100,8),NDAMP(50),NPN(50),NSF(8),NG(8),NN9(50,9),NOW(100)
      COMMON NE,M,NNOD,NRBN,NP,NO,NNQDT,NC
C
      SETTING ELEMENTS OF DAMPING VECTOR EQUAL TO ZERO
      DO 40 I=1,NRBN
      RD(I)=0.00
      RDSS(I)=0.00
40  RDS(I)=0.00
C
C
      FORMULATION OF THE VECTORS RD,RDS, AND RDSS BY THE 4 POINT GAUSS
      QUADRATURE NUMERICAL INTEGRATION SCHEME. VECTOR RD REPRESENTS
      ELEMENTS ON THE MAIN DIAGONAL, VECTOR RDS REPRESENTS THE FIRST OFF
      DIAGONAL ELEMENTS, AND VECTOR RDSS REPRESENTS THE SECOND OFF
      DIAGONAL ELEMENTS
      DO 42 J=3,NRBN,2
      K=J-1
      I=J-2
      II=NDAMP(I)
      JJ=NDAMP(J)
      KK=NDAMP(K)
      DO 42 L=1,4
      GX=GA(L)

```



```

SF(1)= .50D0*(1.D0-GX)*(-GX)
SF(2)= 1.D0*(1.D0-GX**2)
SF(3)= .50D0*(1.D0+GX)*(GX)
XD(1)=-.50D0+GX
XD(2)=-2.D0*GX
XD(3)=+.50D0+GX
XDIF=XD(1)*X(II)+XD(2)*X(KK)+XD(3)*X(JJ)
YDIF=XD(1)*Y(II)+XD(2)*Y(KK)+XD(3)*Y(JJ)
SDIF=DSQRT(XDIF**2+YDIF**2)
YY=SF(1)*Y(II)+SF(2)*Y(KK)+SF(3)*Y(JJ)
COEF=YY*SDIF*GH(L)/SOUND
RD(I)=RD(I)+COEF*SF(1)*SF(1)
RD(K)=RD(K)+COEF*SF(2)*SF(2)
RD(J)=RD(J)+COEF*SF(3)*SF(3)
RUS(I)=RDS(I)+COEF*SF(1)*SF(2)
RDS(K)=RDS(K)+COEF*SF(2)*SF(3)
RDS(I)=RDS(I)+COEF*SF(1)*SF(3)

```

42 ADDING SPHERICAL CONTRIBUTION TO THE MATRIX H

C
C
C

```

R=Y(1)
DO 44 I=1,NRBN
II=NDAMP(I)
S=RD(II)*SOUND/R
H(II,II)=H(II,II)+S
44 DO 46 I=2,NRBN
J=I-1
JJ=NDAMP(J)
II=NDAMP(I)
S=RDS(II)*SOUND/R
H(JJ,II)=H(JJ,II)+S
46 H(II,JJ)=H(II,JJ)+S
DO 48 I=3,NRBN,2
J=I-2
JJ=NDAMP(J)
II=NDAMP(I)
S=RDS(II)*SOUND/R
H(JJ,II)=H(JJ,II)+S
48 H(II,JJ)=H(II,JJ)+S
RETURN
END

```

SUBROUTINE FORCE

IMPLICIT COMPLEX*16 (D),REAL*8 (A-C,E-H,O-Z)

REAL*8 ITITLE

COMMON DSUM(81,81),DB(81)

C


```

COMMON H(81,81),Q(81,81),B(81),X(81),Y(81),HE(8,8),QE(8,8)
COMMON RD(50),RDS(50),RDSS(50),XM(8),YM(8),AMP(25)
COMMON GA(4),GH(4),SF(8),XD(8),YD(8),XSD(8),YSD(8),AJ(2,2),BL(3,3)
COMMON SOUND,FLDEN,OMEGA,OINCR
COMMON NN(100,8),NDAMP(50),NPN(50),NSF(8),NG(8),NN9(50,9),NOW(100)
COMMON NE,M,NNOD,NRBN,NP,NO,NNODT,NC

SETTING ALL ELEMENTS OF THE PISTON FORCING FUNCTION EQUAL TO ZERO

DO 50 I=1,NNOD
50 B(I)=0.00

FORMULATION OF THE FORCING FUNCTION USING THE 4 POINT
GAUSS QUADRATURE NUMERICAL INTEGRATION SCHEME

DO 52 J=3,NP,2
DO 51 LC=1,3
DO 51 LD=1,3
51 BL(LC,LD)=0.00
   K=J-1
   I=J-2
   II=NPN(I)
   KK=NPN(K)
   JJ=NPN(J)
   ED=DSQRT((Y(II)-Y(JJ))**2 + (X(II)-X(JJ))**2)
   DO 54 L=1,4
   GX=GA(L)
   SF(1)= 500*(1.00+GX)*(GX)
   SF(2)= 1.00*(1.00-GX**2)
   SF(3)= 500*(1.00-GX)*(-GX)
   YY=SF(1)*Y(II)+SF(2)*Y(KK)+SF(3)*Y(JJ)
   COEF=(ED/2.00)*YY*GH(L)
   DO 54 LA=1,3
   DO 54 LB=1,3
54 B(LA,LB)=BL(LA,LB)+COEF*SF(LA)*SF(LB)
   B(II)=B(II)+BL(1,1)*AMP(I)+BL(1,2)*AMP(K)+BL(1,3)*AMP(J)
   B(KK)=B(KK)+BL(2,1)*AMP(I)+BL(2,2)*AMP(K)+BL(2,3)*AMP(J)
   B(JJ)=B(JJ)+BL(3,1)*AMP(I)+BL(3,2)*AMP(K)+BL(3,3)*AMP(J)
DO 56 I=1,NP
56 B(II)=B(II)*FLDEN
   II=NPN(I)
RETURN
END

```



```

C      SUBROUTINE PROB
      IMPLICIT COMPLEX*16 (D), REAL*8 (A-C, E-H, O-Z)
      REAL*8 ITITLE
      COMMON DSUM(81,81), DB(81)
      COMMON H(81,81), Q(81,81), B(81), X(81), Y(81), HE(8,8), QE(8,8)
      COMMON RD(50), RDS(50), RDSS(50), XM(8), YM(8), AMP(25)
      COMMON GA(4), GH(4), SF(8), XD(8), YD(8), XXD(8), YYD(8), AJ(2,2), BL(3,3)
      COMMON SOUND, FLDEN, OMEGA, GINCR
      COMMON NN(100,8), NDAMP(50), NPN(50), NSF(8), NG(8), NN9(50,9), NOW(100)
      COMMON NE, M, NNOD, NRBN, NP, NO, NNGDT, NC
      DIMENSION ABPRES(100), PHASE(100)

      INTRODUCING INITIAL VALUE OF OMEGA AND SUBSEQUENT VALUES

      DO 70 KOUNT=1, NO
      WRITE(6,600) OMEGA
      FORMAT('1, VALUE OF OMEGA FOR THIS RUN=', F10.5)
600
      SETTING VALUES OF DSUM EQUAL TO ZERO

      DO 60 I=1, NNOD
      DO 60 J=1, NNOD
      DSUM(I,J)=(0.00,0.00)
60

      SUMMING ALL PROBLEM COMPONENTS

      DO 62 I=1, NNOD
      DO 62 J=1, NNOD
      DSUM(I,J)=DSUM(I,J) + H(I,J) - ((OMEGA**2)*Q(I,J))
62

      II=NDAMP(I)
      DSUM(II,II) = DSUM(II,II) + DCMLPX(0.00, (OMEGA*RD(II)))
63
      I=J-1
      II=NDAMP(I)
      JJ=NDAMP(J)
      DSUM(II,JJ)=DSUM(II,JJ) + DCMLPX(0.00, (OMEGA*RDS(I)))
      DSUM(JJ,II) = DSUM(JJ,II) + DCMLPX(0.00, (OMEGA*RDS(I)))
64
      DO 65 J=3, NRBN, 2
      I=J-2
      II=NDAMP(I)
      JJ=NDAMP(J)
      DSUM(II,JJ)=DSUM(II,JJ) + DCMLPX(0.00, (OMEGA*RDSS(I)))
      DSUM(JJ,II)=DSUM(JJ,II) + DCMLPX(0.00, (OMEGA*RDSS(I)))
65

      MULTIPLICATION OF THE FORCING FUNCTION VECTOR B BY OMEGA SQUARED
      AND STORING IN COMPLEX VECTOR DB

```



```

C
DO 66 I=1,NNOD
DB(I)=DCMPLX(B(I)*(OMEGA**2),0.00)
CONTINUE
66
C
C
C SOLVING PROBLEM USING SUBROUTINE CSIMEQ
CALL CSIMEQ (DSUM,DB,NNOD)
WRITE(6,610)
FORMAT('0:',ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE')
610
WRITE(6,611)
FORMAT('0:',(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)')
611
DO 72 I=1,NNOD
ABPRES(I)=0.00
72
PHASE(I)=0.00
K=0
DO 68 I=1,NNODT
DO 69 J=1,NE
JJ=NN9(J,9)
IF(JJ.EQ.1) GO TO 68
69
CONTINUE
K=K+1
ABPRES(I)=CDABS(DB(K))
A=DB(K)
AA=DB(K)*(0.00,-1.00)
PHASE(I)= 57.295779513100 * DATAN2(AA,A)
CONTINUE
68
I=1
J=9
WRITE(6,612)(ABPRES(K),K=I,J)
612
FORMAT('//9(5X,F9.5)')
WRITE(6,613)(PHASE(K),K=I,J)
613
FORMAT(9(5X,F9.3))
I=I+9
J=J+9
IF(J.LT.82) GO TO 615
C
C
C ACOUSTIC IMPEDANCE AND ENERGY BALANCE
WRITE(6,801)
FORMAT('0:',ACOUSTIC IMPEDANCE')
801
Z=0.00
ZZ=0.00
DO 80 J=1,NP
JJ=NPJ(J)
Z=Z+B(JJ)*(DB(JJ)*(0.00,-1.00))
ZZ=ZZ+B(JJ)*DB(JJ)
80
ZREAL=-2.00*3.141592653600*Z/(OMEGA*FLDEN)

```



```

ZIMAG=2.D0*3.1415926536D0*ZZ/(OMEGA*FLDEN)
TENG1=-Z*OMEGA/(2.D0*FLDEN)
WRITE(6,803) ZREAL
FORMAT(0,5X,REAL=' ',F16.8)
803
WRITE(6,804) ZIMAG
FORMAT(0,5X,IMAG=' ',F16.8)
804
WRITE(6,814)
FORMAT(0,5X,PROBLEM ENERGY BALANCE')
814
WRITE(6,810) TENG1
FORMAT(0,5X,AVG POWER IN=' ',F16.8)
810
ENGO=0.D0
DO 84 I=1, NRBN
II=NDAMP(I)
ENGO=ENGO+CDABS(DB(II))*RD(II)*CDABS(DB(II))
84
DO 86 I=2, NRBN
JJ=I-1
JJ=NDAMP(JJ)
II=NDAMP(II)
ENGO=ENGO+CDABS(DB(II))*RDS(JJ)*CDABS(DB(JJ))*2.D0
86
DO 88 I=3, NRBN, 2
JJ=I-2
JJ=NDAMP(JJ)
II=NDAMP(II)
ENGO=ENGO+CDABS(DB(II))*RDSS(JJ)*CDABS(DB(JJ))*2.D0
88
TENG0=ENGO/(2.D0*SOUND*FLDEN)
WRITE(6,812) TENG0
FORMAT(0,5X,AVG POWER OUT=' ',F15.8)
812
OMEGA = OMEGA + OINCR
70 RETURN
END

```

SUBROUTINE NODE(NXDIV, NYDIV)

```

IMPLICIT COMPLEX*16 (D), REAL*8 (A-C, E-H, O-Z)
REAL*8 ITITLE
COMMON DSUM(81,81), DB(81)
COMMON H(81,81), Q(81,81), B(81), X(81), Y(81), HE(8,8), QE(8,8)
COMMON RD(50), RDS(50), RDSS(50), XM(8), YM(8), AMP(25)
COMMON GA(4), GH(4), SF(8), XD(8), YD(8), XXD(8), YYD(8), AJ(2,2), BL(3,3)
COMMON SOUND, FLDEN, OMEGA, OINCR
COMMON NN(100,8), NDAMP(50), NPN(50), NN9(50,9), NOW(100)
COMMON NE, M, NNOD, NRBN, NP, NO, NNODT, NC

```

ASSIGNING NODE NUMBERS TO ELEMENTS

WRITE(6,126)


```

126 FORMAT('0','ASSIGNMENT OF NODE NUMBERS TO ELEMENTS')
I1=1
I2=NXDIV
L=4*NXDIV+3
K=2*NXDIV+2
J=1
DO 30 KK=1,NYDIV
DO 30 I=I1,I2
NN9(I,1)=L+1
NN9(I,2)=L+2
NN9(I,3)=L+2
NN9(I,4)=K+2
NN9(I,5)=J+1
NN9(I,6)=J
NN9(I,7)=K
NN9(I,8)=K+1
NN9(I,9)=K+1
L=L+2
K=K+2
J=J+2
30 L=L+2*NXDIV+2
J=J+2*NXDIV+2
K=K+2*NXDIV+2
I1=I1+NXDIV
I2=I2+NXDIV
32 WRITE(6,127)
127 FORMAT('0','ELEMENT NO',5X,'NODE NUMBERS (8 NODED ELEMENTS)')
DO 31 I=1,NNODT
31 NOW(I)=0
I1=0
DO 40 I=1,NNODT
DO 42 J=1,NE
JJ=NN9(J,9)
IF(JJ.EQ.1) GO TO 40
42 CONTINUE
I=I+1
NOW(I)=I
40 CONTINUE
DO 34 I=1,NE
DO 35 J=1,M
I1=NN9(I,J)
35 NN(I,J)=NCW(I1)
34 WRITE(6,128)I,((NN(I,J),J=1,M)
128 FORMAT(3X,I2,11X,8I5)
RETURN
END

```



```

SUBROUTINE XYFORM(NXDIV,NYDIV)
IMPLICIT COMPLEX*16 (D),REAL*8 (A-C,E-H,O-Z)
REAL*8 ITITLE
REAL*8 LABEL/8H
COMMON DSUM(81,81),DB(81)
COMMON H(81,81),Q(81,81),B(81),X(81),Y(81),HE(8,8),QE(8,8)
COMMON RD(50),RDS(50),RDSS(50),XM(8),YM(8),AMP(25)
COMMON GA(4),GH(4),SF(8),XD(8),YD(8),XSD(8),YSD(8),BL(3,3)
COMMON SOUND,FLDEN,OMEGA,OINCR
COMMON NN(100,8),NDAMP(50),NPN(50),NSF(8),NG(8),NN9(50,9),NOW(100)
COMMON NE,M,NNOD,NRBN,NP,NO,NNODT,NC
REAL*4 XDRAW,YDRAW
COMMON /ITITLE/ITITLE(12)
DIMENSION XVAL(25),YVAL(25),XDRAW(25),YDRAW(25),XF(100),YF(100)

```

CALCULATE X AND Y COORDINATES

```

XINCR=1.DO/NXDIV
YINCR=1.DO/NYDIV
NX=2*NXDIV+1
NY=2*NYDIV+1
DO 10 I=1,NX
  XVAL(I)=0.DO
DO 11 I=1,NY
  YVAL(I)=0.DO
DO 12 I=1,NX
  J=I-1
  XVAL(I)=XVAL(J)+XINCR
  YVAL(I)=1.DO
DO 19 I=2,NY
  J=I-1
  YVAL(I)=YVAL(J)-YINCR
19 WRITE(6,121)
121 FORMAT(0,'VALUES OF XI AND ETA FOR WHICH X AND Y COORDINATES WILL BE CALCULATED')
WRITE(6,126)
126 FORMAT(0,'5X','VALUES OF XI',5X,'VALUES OF ETA')
125 WRITE(6,125)(I,XVAL(I),YVAL(I),I=1,NX)
  FORMAT(12,5X,F10.5)
LL=1
DO 16 I=1,NY
DO 27 J=1,NX
  GX=XVAL(J)
  GY=YVAL(I)
  SF(1)=.25D0*(1.DO-GX)*(1.DO-GY)*(-GX-GY-1.DO)

```



```

SF(2)= -50D0*(1.D0-GX**2)*{(1.D0-GY)
SF(3)= -25D0*(1.D0+GX)*{(1.D0-GY)*{(+GX--GY-1.D0)
SF(4)= -50D0*(1.D0+GX)*{(1.D0-GY**2)
SF(5)= -25D0*(1.D0+GX)*{(1.D0-GY)*{(+GX+GY-1.D0)
SF(6)= -50D0*(1.D0-GX**2)*{(1.D0+GY)
SF(7)= -25D0*(1.D0-GX)*{(1.D0+GY)*{(-GX+GY-1.D0)
SF(8)= -50D0*(1.D0-GX)*{(1.D0-GY**2)
XF(LL)=0.D0
YF(LL)=0.D0
DO 28 K=1,M
XF(LL)=XF(LL)+SF(K)*XM(K)
YF(LL)=YF(LL)+SF(K)*YM(K)
CONTINUE
WRITE(6,127) VALUES OF X AND Y'
127 FORMAT(6,120)
120 FORMAT('O',(X COORDINATES ON TOP-Y COORDINATES ON BOTTOM)')
I=1
J=2*NXDIV+1
N={2*NXDIV+1}*{(2*NXDIV+1)
WRITE(6,128)(L,L=I,J)
FORMAT('O',8X,I2,8(11X,I2))
128 WRITE(6,122)(XF(L),L=I,J)
124 WRITE(6,123)(YF(L),L=I,J)
122 WRITE(6,123)(YF(L),L=I,J)
FORMAT(9(5X,F8.5))
I=I+(2*NXDIV+1)
J=J+(2*NXDIV+1)
IF(J.LT.(N+1)) GO TO 124
PLOT CORNER NODES AS POINTS ON A GRAPH
I=1
J=2*NXDIV+1
II=1
JJ={2*NXDIV+1}*{(2*NXDIV)+1}
NNX=NXDIV+1
NNY=NYDIV+1
KK=4*NXDIV+2
LL=1
DO 15 L=I,J,2
XDRAW(LL)=XF(L)
YDRAW(LL)=YF(L)
LL=LL+1
129 DO 15 L=L+1
K=2
IF(I.EQ.1) K=1
CALL DRAW(NNX,XDRAW,YDRAW,K,O,LABEL,ITITLE,O,O,O,O,5,7,O,LAST)
I=I+(4*NXDIV+2)
15

```



```

406 A(J,K)=A(K,J)
404 B(J)=B(J)-A(J,I)*B(I)
401 CONTINUE
      BACK SUBSTITUTION
403 I1=I
      I=I-1
      IF(I) 411,411,412
412 DO 413 J=I1,N
413 B(I)=B(I)-A(I,J)*B(J)
411 GO TO 403
      RETURN
411 END

```

C
C
C

APPENDIX C

ISO-PARAMETRIC ELEMENTS

The family of iso-parametric elements, recently developed by Zienkiewicz, Irons, and others, has brought new versatility and power to the finite element method. The special usefulness of these elements is that they may be shaped to conform to curvilinear boundaries. Attention is confined here to two-dimensional parabolic elements. A full description of the entire family is given in Ref. [2].

Consider the rectangular element of Figure 5. The element has four corner nodes and four midside nodes, numbered as shown. Points in the interior, or on the boundary, may be located in the local (ξ, η) coordinate system. The dependent variable, pressure p in the present application, is defined at any point by Eq. (3), repeated here

$$p = \sum_{i=1}^8 N_i p_i \quad (3)$$

where

$$N_i = \frac{1}{4}(1 + \eta\eta_i)(1 + \xi\xi_i)(\eta\eta_i + \xi\xi_i - 1)$$

for corner node i , and

$$N_i = \frac{1}{2}(1 - \eta^2\xi_i^2 - \xi^2\eta_i^2)(1 + \eta\eta_i + \xi\xi_i)$$

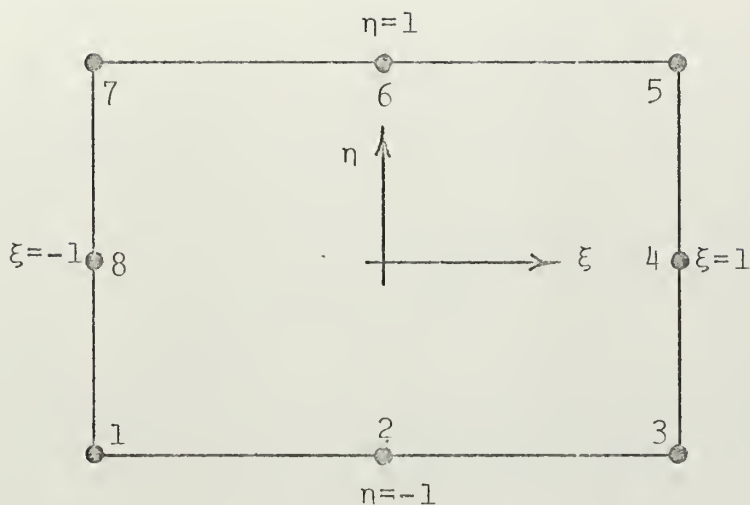


Figure 5. Element Configuration in Local ξ, η Coordinates

for mid-side node i ,

(ξ_i, η_i) are coordinates of node i ,

and p_i is the pressure at node i .

Note that each shape function N_i has the value unity at node i , and is zero at all other nodes.

A curvilinear element, such as that of Figure 6, may be produced by mapping the rectangle of Figure 5 into the x, y plane by the relations

$$x = \sum_{i=1}^8 N_i x_i, \quad y = \sum_{i=1}^8 N_i y_i \quad (19)$$

where (x_i, y_i) are the cartesian coordinates of node i .

The shape of the element in the x, y plane is uniquely determined by specifying the coordinates (x_i, y_i) for the

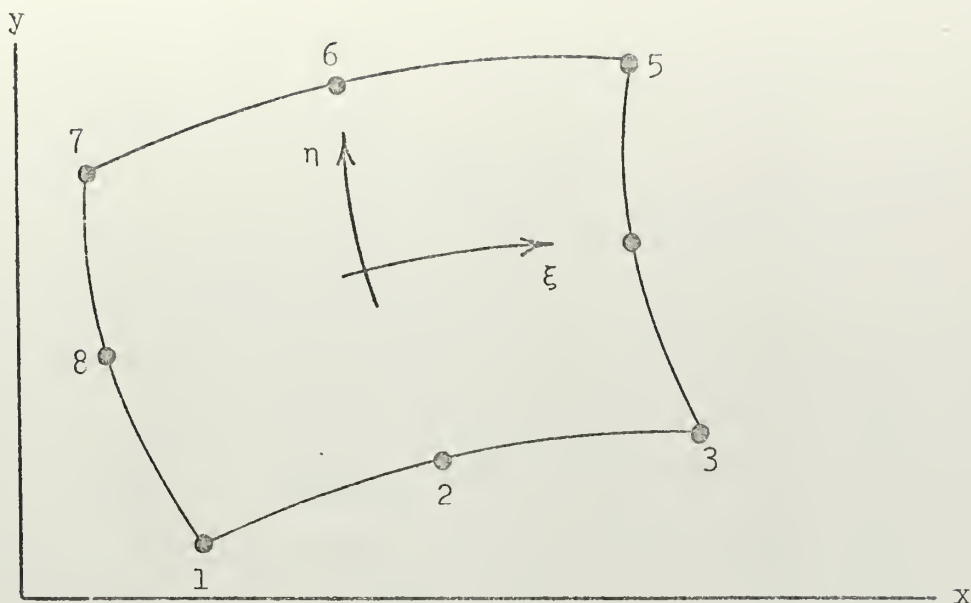


Figure 6. Curvilinear Elements Mapped in Cartesian Coordinates.

eight nodes. The edges of this curvilinear element are parabolic.

Adjacent elements "share" the same three nodes at their interface. When mapped to cartesian coordinates, these two adjacent elements then have the same curvilinear shape and the same x and y coordinates at nodes. This provides continuity in the pressure distribution across element boundaries.

The region developed for the present study necessitates the use of iso-parametric elements to provide the circular non-reflection boundary. This boundary is constructed using parabolic arc segments which are mapped by use of the shape functions to "fit" a quarter circle boundary in cartesian coordinates.

APPENDIX D

FINITE ELEMENT MESH GENERATOR

Two subroutines, included in the computer program of Appendix B, should prove useful in a variety of finite element problems. Together, these subroutines subdivide the region, systematically assign element numbers and node numbers, calculate nodal coordinates, and produce a computer plot of the region to be studied. As an example, consider the region of Figure 7.

The region is taken to be one large iso-parametric element (a mapping of the rectangle of Figure 5). Its shape is prescribed by specifying the (x,y) coordinates of the

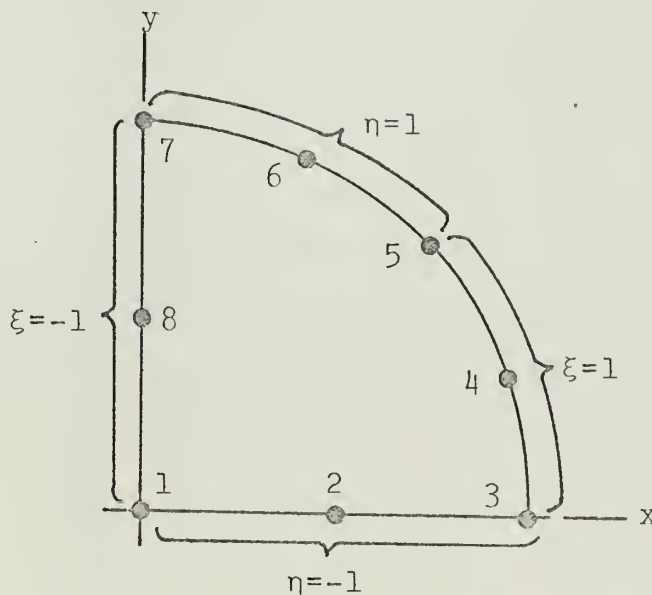


Figure 7. Basic Geometry of Finite Element Region.

eight nodes. Subdivision of the element can then be accomplished at predetermined values of ξ and η by using Equation (19).

Subroutine NODE (Appendix B) requires two input parameters, the number of divisions desired in the x direction and the number of divisions desired in the y direction. These parameters are used to calculate the increment values of ξ and η for subdividing the element into smaller elements. The subroutine starts numbering global nodes at the upper left hand corner ($\xi=-1$, $\eta=1$) then adds increment values of ξ and assigns global node numbers from left to right on the boundary $\eta=1$. Similarly, after incrementing η , the second row of global node numbers is assigned, etc. Global node numbers are assigned to all corner nodes, mid-side nodes and the center of each subdivided element (9 nodes per element). The center node provides unique identification for the element and simplifies the printing of computer output in a readable form. Subsequently all nodes are renumbered, omitting the center nodes.

The cartesian coordinates corresponding to the global node numbers are calculated by a similar process in the subroutine XYFORM and are used in the program to plot the finite element grid. Figure 8 illustrates a computer plot of the finite element mesh generated for a quarter circular region (Figure 7) with an eight foot radius. The number of subdivisions in both the x and y direction is equal to four. The plotting routine produces straight segments

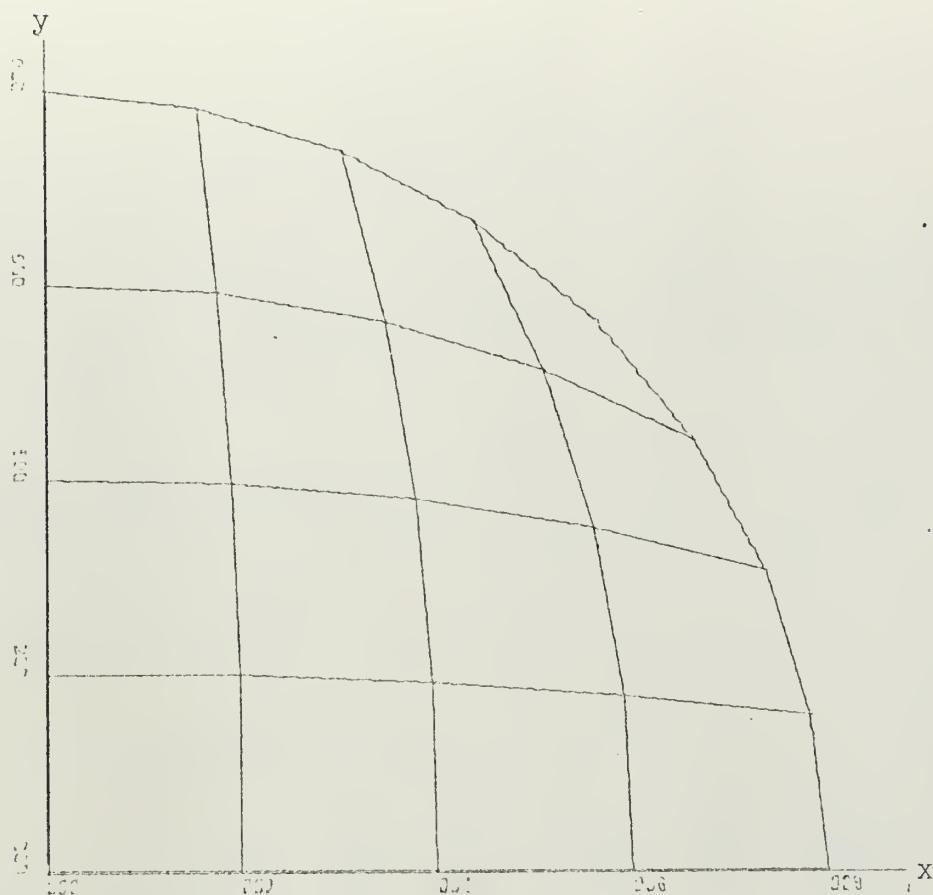


Figure 8. Finite Element Mesh Generated
by Subroutine XYFORM.

between nodes. In this instance, the element boundaries are all parabolic arcs except for those lying on the x and y axes.

APPENDIX E

MATRIX ELEMENT FORMULAS

Individual elements of the matrices [Q], [D], [L], and [H] of the system of ordinary differential equations (Eq. (4)) are given by Ref. [3] as

$$q_{ij} = \frac{1}{c^2} \int_{Re} N_i N_j dR$$

$$d_{ij} = \frac{1}{c} \int_{Se} N_i N_j dS$$

$$l_{ij} = \int_{Se} N_i' N_j dS$$

$$h_{ij} = \iint_{Re} \left[\frac{\partial N_i}{\partial x} \cdot \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \cdot \frac{\partial N_j}{\partial y} + \frac{\partial N_i}{\partial z} \cdot \frac{\partial N_j}{\partial z} \right] dR$$

where Re and Se are the element region and external boundary respectively.

The shape functions N_i , N_i' are specified in terms of local coordinates and it is advantageous to perform the integrations in these coordinates. Gaussian quadrature is employed. Details of the procedure are given in Ref. [2].

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The generation and propagation of small amplitude acoustic waves in a homogeneous, loss-free, compressible fluid is studied by the finite element method. A diaphragm mounted in an infinite rigid baffle generates acoustic waves in a semi-infinite fluid region. Steady-state pressure distribution is found for a hemispherical region with boundary reflection suppressed through use of a radiation condition. The computer program developed for the purpose utilizes iso-parametric finite elements with curvilinear boundaries. Incorporated in the program is a versatile mesh generator which minimizes the quantity of input data. Acceptable agreement with analytic results is obtained when there are at least four elements per wave-length.

| 14
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| | ROLE | WT | ROLE | WT | ROLE | WT |
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| ACOUSTIC WAVES | | | | | | |
| RADIATION IMPEDANCE | | | | | | |

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